OPTIMAL UTILIZATION OF COASTAL UNONFINED AQUIFERS

الاستخدام الأمثل للخزانات الجوفية الساحلية غير المحصورة

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خلاصة

يهـتم هذا البحث بالحصول على امثل استخدام الخزانات الجوفية الساحلية الغير محصورة. تلك المشكلة معقدة وغـير خطـية ومقـيدة. ولذلك تم استخدام الطريقة الحسابية الجينية للوصول لأقصى منفعة. وقد استخدم الحل التحليلي لتمثيل سريان المياه الجوفية وذلك تحت ظروف سريان مستقر داخل خزان جوفي متجانس. واستخدمت الـنظرية الانعكاسـية مع مبدأ التركيب (التجميع التراكمي) للحصول على الحل التحليلي. وقد تم كتابة برنامج للحاسوب بطريقة الفورتران لحل المشكلة تحت الدراسة. وأخيرا طبق ذلك البرنامج على الخزان الجوفي أسفل مدينة ميامى بشمال أسبانيا وذلك لتوافر البيانات الخاصة بهذا الخزان.

ABSTRACT

This research concerns with optimal utilization of unconfined coastal aquifers under different constraints. This problem is a complicated, nonlinear, and constrained one. Therefore, the Genetic Algorithm nonlinear optimization method was selected to handle the present problem. Analytical solution was applied for the groundwater flow equation under assumption of steady sharp interface in homogenous aquifer. Image theory and the superposition principle were the main tools used through the analytical procedure. A Fortran program was written to apply the mathematical concepts of the problem. The method was applied to aquifer system underlain the City of Miami Beach at north of Spain.

Key Words: *Coastal Unconfined Aquifer- Well System- Genetic Algorithm- Analytical Solution- Image Theory- Superposition Principle- Optimal Strategy.*

1-INTRODUCTION

Coastal aquifers serves as major sources for freshwater supply in many countries, especially in arid and semiarid regions. Many coastal areas are heavily urbanized, a fact that makes the need for fresh water more acute. Coastal aquifers are highly sensitive to disturbances. Inappropriate management may lead to their destruction as a source for freshwater due to the intrusion of salt water from the sea [1]. In Egypt, there is a growth of population in the Nile Delta region, consequently a continuous increase of pumping rates from the coastal aquifer at the Nile Delta can be observed. This aquifer is a leaky confined one, from the eastern side (at the Suez Canal) to the middle of the Delta (between Cairo and lake El-Burrullus). The western part of the Nile Delta aquifer is an unconfined phreatic aquifer [2]. This research concerns with optimal utilization (maximum allowed pumping rates of freshwater under

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different constraints) of phreatic coastal aquifers. Consequently the procedure followed within this work can be applied to the western part of the Nile Delta aquifer.

In 2002, Sherif studied six different scenarios of pumping rates from the leaky part of the Nile Delta aquifer [3]. He concluded that any additional pumping should be practiced from the middle Delta. Indeed, only six strategies can not represent the numerous amount of alternatives that can be applied. Therefore, a powerful optimization technique must be applied to catch the best strategy. Optimal pumping from coastal aquifers is a complicated problem, due to the fact that the optimization domain is discontinuous with multiple local minimums [4, 5]. So, gradient methods are going to fail in achieving the global minimum (best strategy). The solution is always dependent on the starting point (first initial guess of pumping rates) [6]. In the other hand, stochastic optimization is suitable to handle the present problem [7]. This is the reason for selecting genetic algorithm as the optimization tool in this work. This method starts with a population of initial guesses/solutions that provide a good insight about the complexity of the optimization domain.

In planning analyses, a simulation model must be used to investigate how pumping alternatives affect the movement of saltwater. Due to the sea water intrusion mechanism, freshwater zone, mixing zone, and sea water flow zone occur in coastal aquifers. Two approaches of modeling the seawater intrusion phenomenon are available.

In one approach, that adopted in this work, a sharp interface without any mixing zone is assumed to separate the freshwater and seawater zones within the aquifer. This approach is completely satisfactory if the width of the transition zone is relatively small when compared with the thickness of the aquifer [1]. In case of steady flow, the simplest sharp interface formulation is obtained, freshwater zone is only considered and seawater is regarded as static. The flow system is described using only the freshwater equation. The key modeling assumption is known as the Ghyben-Herzberg approximation [1]. For homogenous and isotropic aquifers with straight boundaries analytical solutions can be used to estimate the steady location for the sharp interface [1, 4].

The more realistic approach (second one) for modeling seawater intrusion considers a transition zone between freshwater and saltwater, which requires simultaneous solution of the governing equations of fluid flow and solute transport. Such an approach leads to density-dependent transport models. To simulate areal problems, three dimensional modeling must be applied [8, 9], that requires long computation time and storage space. So, the second approach is not suitable to simulate seawater intrusion through the optimization process.

The purpose of this study is to find the optimal pumping strategy that can be applied on a steady phreatic aquifer, satisfying different constraints.

2- PROBLEM FORMULATION

For the phreatic aquifer shown in Fig.1, the following assumptions were taken into consideration: 1) the sharp interface model was used, 2) the Dupuit's hydraulic assumption was employed to vertically integrate the flow equation, reducing it from three-dimensional geometry to two-dimensional, 3) the aquifer storativity was ignored such that the governing equation became time independent, 4) the Ghyben-Herzberg assumption of stagnant salt water was utilized to interpret the interface location, 5) the wells fully penetrate the aquifer thickness , 6) the impervious bed of the aquifer was considered horizontal, and 7) hydraulic conductivity was assumed constant throughout the studied domain.

The governing equations that describe a phreatic aquifer can be written as follows[10]:

$$\frac{\partial}{\partial x}\left(Kh\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(Kh\frac{\partial h}{\partial y}\right)=$$

$$\sum_{i=1}^{nw}\delta(x-x_i)Q_i\delta(y-y_i) \textit{ for zone I} \quad \text{(1-a)}$$

$$\frac{\partial}{\partial x}\left(K\frac{(h-d)}{\varepsilon}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{(h-d)}{\varepsilon}\frac{\partial h}{\partial y}\right)$$

$$=\sum_{i=1}^{nw}\delta(x-x_i)Q_i\delta(y-y_i) \textit{ for zone II} \quad \text{(1-b)}$$

where, h is the freshwater piezometric head *(L)*, $\varepsilon=(\rho_s-\rho_f)/\rho_s$, ρ_s and ρ_f are saltwater and freshwater densities respectively *(M/L³)*, d is elevation of mean sea level above the impervious bed of the aquifer *(L)*, x *and* y are rectangular coordinates *(L)*, Q_i is the discharge of the well i, which is located at coordinates (x_i, y_i), negative for discharging and positive for recharging (L^3/T), nw represent number of wells in the studied domain, $\delta(z)$ is the Dirac function equal to 1 if z equal to *0.0*, otherwise equal to zero, and K is the hydraulic conductivity *(L/T)*. Zone *I* contains only freshwater, while zone II contains both fresh and sea water.

In 1976, Strack described how the nonlinear equations (1-a, b) can be reformulated in linear form, without any approximation, [4, 10]. The dependent variable h must be replaced with the potential function ϕ. Hence equations (1-a, b) can be rewritten in the following form:

$$\frac{\partial}{\partial x}\left(K\frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial \phi}{\partial y}\right)=$$

$$\sum_{i=1}^{nw}\delta(x-x_i)Q_i\delta(y-y_i)$$

$$\textit{for zone I, zone II} \quad (2)$$

where, $\phi=(h^2-s\,d^2)/2$ *for zone I*, $\phi=(h-d)^2/2\varepsilon$ *for zone II*, $s=\rho_s/\rho_f$, and the two zones separated at $\phi_{toe}=s(s-1)d^2/2$. Boundary conditions for the studied aquifer can be represented as:

$$\partial h/\partial n=\partial\phi/\partial n=0.0$$

$$\textit{on sides ij, lk [Neumman B.C.]} \quad (3)$$

$$h_o=d+\overline{BC} \quad \textit{or}$$

$$\phi=(h_o-d)^2/2\varepsilon=\overline{BC}^2/2\varepsilon$$

$$\textit{on side jk} \quad \textit{[Dirichlet B.C.]} \quad (4)$$

$$h\partial h/\partial n=\partial\phi/\partial n=q_u/K$$

$$\textit{on side il} \quad \textit{[Neumman B.C.]} \quad (5)$$

where, q_u represents uniform rate of discharge per unit width of the aquifer L^2/T, and $\overline{BC}$ is the seepage face for freshwater at sea coast (*L*). It must be noticed that the seepage faces around the wells are ignored within the range of the present study.

3- ANALYTICAL SOLUTION

The above problem can be solved analytically for homogenous isotropic soil with horizontal bed. This can be achieved by using the image method together with the superposition principle. Figure 2 shows the principles of the image theory for one pumping well w located at coordinates (x_w, y_w) [1,10]. The location of a well which acts as an image to real well w is a mirror image, with respect to the straight line boundary. For a constant potential head boundary (*jk*) the image of the pumping well performance. The well discharges freshwater only if *x-well* ≥ *x-ϕ toe*.

To find the location of *ϕ toe* a nonlinear optimization method must be applied. In this work the steepest decent method was used [12, 13, 14]. Since the potential is continuous and smooth, this gradient method was quite efficient.

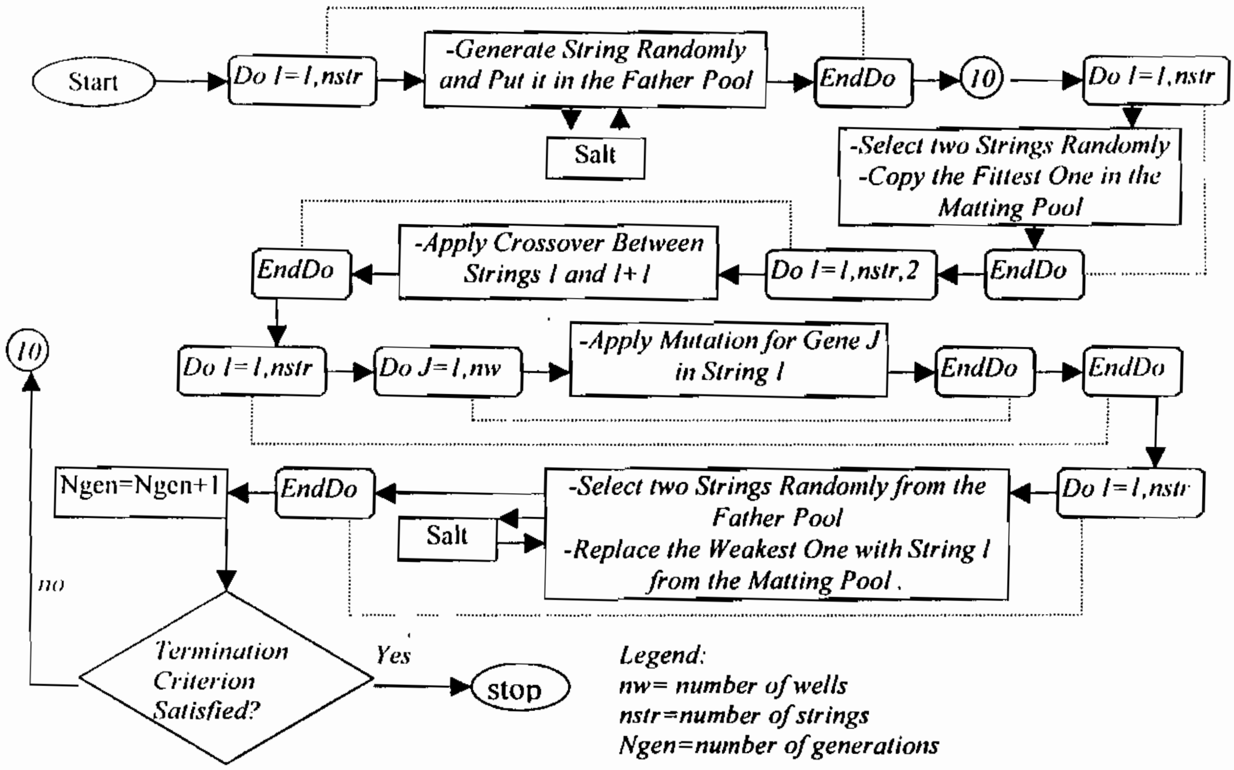

Figure 3 Flowchart for Genetic Algorithm.

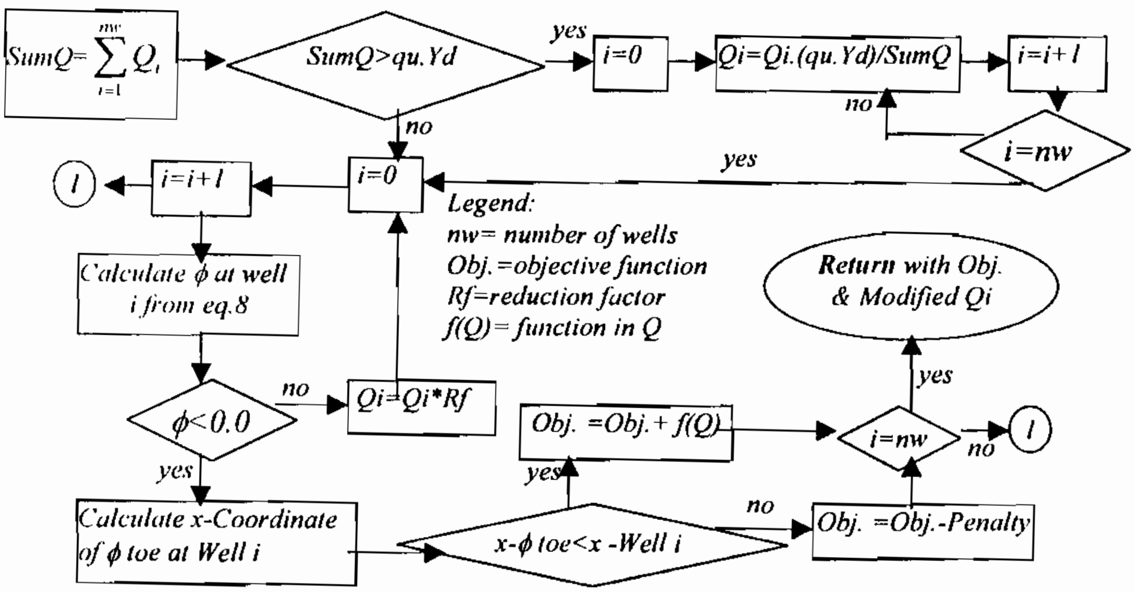

Figure 4 Flowchart for Sub-program Salt.

5- MODEL VERIFICATION

A Fortran program was written to apply the previous concepts. To verify that code and the methodology, the program has to be tested against analytical problem. Cheng, [4], gave an analytical solution for maximum allowable pumping rates from two wells parallel to the coast. The two wells are *1000m* from the coast and 500m apart from each other. Figure 5 shows the geometry of the problem. The following parameters were used $K=100m/d$, $q_u=0.6m \; m^2/d$, $d=14m$, $\rho_s=1.025$ gm/cm^3, and $\rho_f=1.0\ gm/cm^3$.

Maximum allowable pumping rate from each well according to the analytical solution is equal to $423.4\, m^3/d$, with total magnitude of pumping rate equal to 846.8 m^3/d. While total pumping rate using the present program was equal to 850.6 m^3/d, with pumping rate equal to 421.9 *m3/d* for one well and 428.7 *m3/d* for the other one. Relative difference between total pumping rate was less than 0.005%. This minor discrepancy may be due the unreal assumption, that is necessary to complete the analytical solution, of equivalent pumping rates form the two wells.

6- NUMBER OF MIRRORS AGAINST ACCURACY OF THE ANALYTICAL SOLUTION

The present problem includes two opposite impervious boundaries *ij*, *kl* in Figs. 1, 2. This produces an infinite number of mirrors, which creates multiple image wells, within the analytical solution, Eq. 6. So, it is necessary to determine the numerical/actual number of mirrors that can be used inside the program, without scarifying the accuracy of the analytical solution. Figure 6, shows relative ratio of ϕ after a specified number of mirrors n, with respect to value of ϕ when $n=1$. The following parameters were considered through the calculations: $Yd=1m$, $yw=0.0m$, $xw=1.0m$, $x=1.0m$, $y=1.0m$. It can be noticed that the relative ratio of ϕ convergences quickly to constant value as n increases. For the following case study n was taken equal to one hundred.

7-CASE STUDY - (MIAMI BEACH, SPAIN)

The optimal pumping rates have been applied here to the unconfined coastal aquifer at the Miami Beach in the north of Spain [5]. This aquifer can be considered as one of the most important water-supply sources for the City of Miami Beach. Lithology of Miami coastal aquifer consists of unconsolidated sediments of Quaternary age, with pore permeability, corresponding to coastal piedmonts and alluvial fans, and is generally single-layered. The sediment consists of clay and gravel, and overlie a blue clay of Pliocene age, which constitutes the effective lower impervious hydrologic boundary.

The hydraulic parameters are estimated to be, width of the aquifer $Yd=5000m$, hydraulic conductivity $K=14m/d$, uniform discharge per unit width of the aquifer $q_u=1.2m \; m^2/d$, depth of the impervious bed from mean sea level $d=30m$, density of sea water $\rho_s=1.025\ gm/cm^3$, and density of fresh water $\rho_f=1.0\ gm/cm^3$ [5].

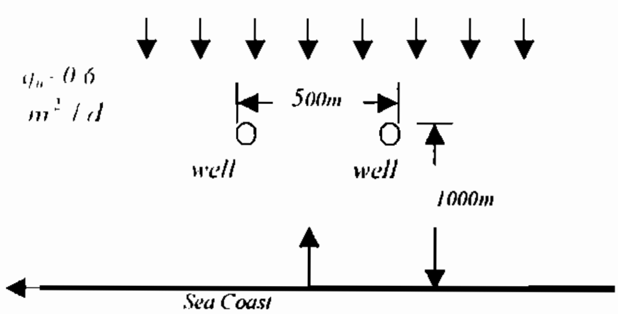

Figure 5. Definition sketch of two-well problem.

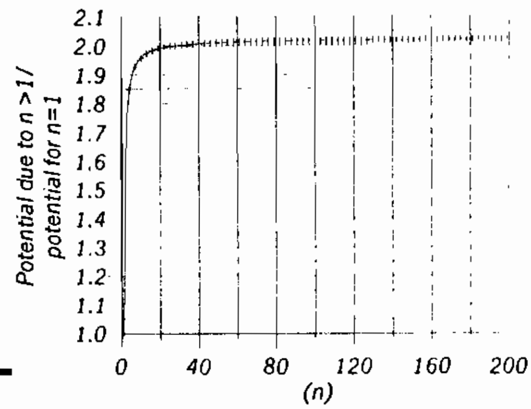

Figure 6. Effect of number of mirrors (n) on accuracy of the analytical solution.

Table 1, Pumping wells input data and results for the unconfined aquifer of Miami Beach.

Input Data						*Results*
(1) Well number	*(2) x Coordinate m*	*(3) Y Coordinate m*	*(4) Maximum pumping rate m^3/d*	*(5) Minimum pumping rate m^3/d*	*(6) Ground level (m)*	*(7) Pumping rate m^3/d from GA*
1	3877.26	4361.68	1200	120	79.798	0.0
2	3825.54	3747.51	1200	120	112.869	0.0
3	3654.91	3390.29	1200	120	110.598	0.0
4	3625.24	2647.67	1000	120	89.0	0.0
5	3507.27	3686.34	1200	120	80.685	0.0
6	3469.01	3899.55	1200	120	78.414	120.0
7	3285.20	4148.47	1200	120	66.077	958.652
8	3160.71	4714.84	1000	120	66.946	491.937
9	3133.13	2592.51	1000	120	85.0	0.0
10	2807.67	960.58	900	120	90.66	0.0
11	2743.80	2705.04	1000	120	70.0	0.0
12	2647.26	758.77	900	120	89.156	0.0
13	2046.90	2495.63	1200	120	65.0	675.1592
14	1322.07	2922.15	300	120	25.0	300.0
15	1245.52	2540.73	300	120	30.0	300.0
16	1077.24	2768.99	300	120	22.0	275.0
17	905.54	2761.39	300	120	19.0	218.6249
18	872.59	4202.28	300	120	20.0	300.0
19	703.79	762.84	300	120	34.0	300.0

The depth of the aquifer perpendicular to the sea coast was taken equal to *4500 m*. Table 1, shows coordinates of different wells pre-existed in the aquifer, their maximum and minimum rates, and the corresponding ground level. The management strategy was settled to maximize the economic benefit from the pumped water and to minimize the cost corresponding of lifting the water. Then the objective function (Obj) can be written as:

$$Obj=\sum_{i=1}^{nw} Q_i\left[B-C(L_i-h_i)\right] - p*\sum_{i=1}^{nw} H(x_i - xtoe_i) \qquad (9)$$

where, B is the economic benefit for one cubic meter of freshwater $\$/m^3$, C is cost of lifting one cubic meter of water for one meter height $\$/m^3$, L_i is ground level (L), h_i is groundwater level (L), p is a constant penalty for intruded wells \$/(intruded well), x_i is the x-coordinate for well i, $xtoe_i$ is the x-coordinate of the sharp interface toe at y-coordinate equals to y-coordinate of well i, and $H(z)$ is the *Heaviside* unit step function equal to 1 if $z > 0$, otherwise equal to zero. The above objective function is constrained between the upper and lower limits of pumping rates for different wells.

The following parameters were used for the GA in this problem, population size = *nstr* = *100*, maximum number of generation =*100*, crossover ratio =*0.75*, mutation ratio = 0.*15*, $B = 0.01\$/m^3$, $C = 0.0002\$/m^3/m$, $p=6.0$ *\$/d* (obtained by iteration), and no seepage face at the sea coast ϕ_n=0.0. Pumping rate from any well must be considered equal to zero if the generated uniform random number is less than 0.3, otherwise ranges between allowable minimum and maximum rates of pumping of that well.

Optimal solution for net benefit was found equal to 23.809 \$/day with total pumping rate equal to 3939.37 m^3/d which was equal to 65.6% of total water enters to the aquifer. In the other hand, Benhachmi [5] found that optimal benefit was equal to 28.95 \$/day with total pumping rate equal to 8929.31 m^3/d, equivalent to 148.8% of total water enters the aquifer. This unrealistic result may be due to the effect of number of mirrors used to represent the analytical solution. Using a smaller number of images over-estimates the potential ϕ through the studied domain, and consequently a false movement for the sea water wedge towards the sea coast can be expected. Pumping rates from different wells are mentioned in column 7 in table 1. Most of pumped water were taken from wells that have low ground level to decrease the lifting cost of pumped water. In this problem near the sea coast there was a region that has a ground water level higher than the ground level. If pumped water is sucked from that region the lifting cost C was taken equal to zero. Figure 7, shows the levels of the ground.

Figure 8, shows convergence rate to optimal solution. It can be noticed that optimal solution for the objective function increases quickly with a decreasing rate as the number of generations increases. GA approaches optimal solution approximately after 50 generations. Figure 9, shows contours of ground water level, the shaded area represents the intruded area and contour line corresponding to $h_{toe} = d.\rho_s/\rho_f$ = 30.70m represents the sharp interface toe.

The above aquifer re-studied again for different strategy, that maximizes the total pumping of freshwater. Consequently different objective function must be introduced, as follows:

$$Obj = \sum_{i=1}^{nw} Q_i - p * \sum_{i=1}^{nw} H(x_i - x\,toe_i) \quad (10)$$

The same parameters used in the first example were reused here, except the penalty was taken equal to $2000\,m^3$/(intruded well). Optimal solution of pumping rates was found equal to $5460.698\,m^3/d$, that represents about 91% of total water enter the aquifer. Different calculated pumping rates can be seen in table 2.

Table 2. Pumping wells output from the unconfined aquifer of Miami Beach with objective function in eq. 10.

(1) Well number	(7) Pumping rate m^3/d from GA
1	533.915
2	528.088
3	479.0104
4	766.197
5	263.562
6	347.909
7	279.974
8	309.141
9	463.289
10	554.829
11	282.89
12	480.68
13	171.23
14	0.0
15	0.0
16	0.0
17	0.0
18	0.0
19	0.0

It can be noticed that most of the pumped freshwater were sucked from wells located far from the sea cost. This was due to the advancing of the sea water wedge inland under excessive withdrawal of freshwater.

Figure10, shows convergence rate of optimal solution with respect to number of generations. Figure 11, exhibits contour lines of groundwater levels when the objective function concerned with optimal rate of total pumping. Salt water intruded a large region of the aquifer due to the excessive pumping of water. Table 2, introduces different pumping rates from the aquifer, most of the sucked freshwater were taken from wells located far from the sea coast. This was due to the intrusion of most wells near to the coast line with sea water.

8- CONCLUSIONS AND RECOMMENDATIONS

In this paper, the use of the genetic algorithm has been introduced with the analytical solution to estimate optimal pumping of fresh water from coastal unconfined aquifers under certain conditions. The analytical solution constrained for homogenous steady ground water with straight boundaries and horizontal impervious bed. From the present study the following points can be concluded:

1) The number of mirrors used to represent the two opposite impervious boundaries *ij, kl* affects the accuracy of the analytical solution. Sufficient number of mirrors must be taken to satisfy the accuracy requirement. Inaccurate values from the analytical solution will overestimate ground water level

and consequently overestimates the optimal solutions.

2) As the generation process optimal solution enhances quickly and approaches approximately its maximum magnitude after fifty generations.
3) The intruded zone and total pumping rate of freshwater are mainly dependent on the objective function and the constraint conditions.
4) For the present study seepage faces at the sea coast and around the pumping wells were ignored which underestimates the actual water table and consequently decreases the optimal solution.

The present procedure can easily be applied for confined coastal aquifers. For more practical domains, e.g., heterogeneous aquifer with curved boundaries and variable thickness, a more sophisticated method like the Finite Element Method must be used instead of the analytical solution.

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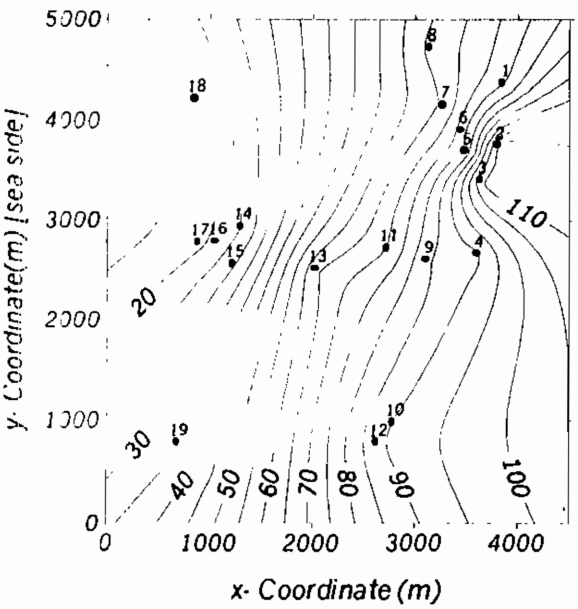

Figure 6 Ground-level and well system.

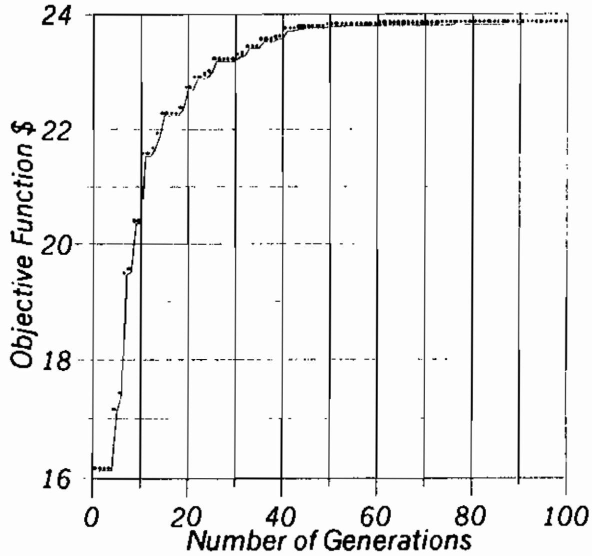

Figure 7 Optimal solution against number of generations.

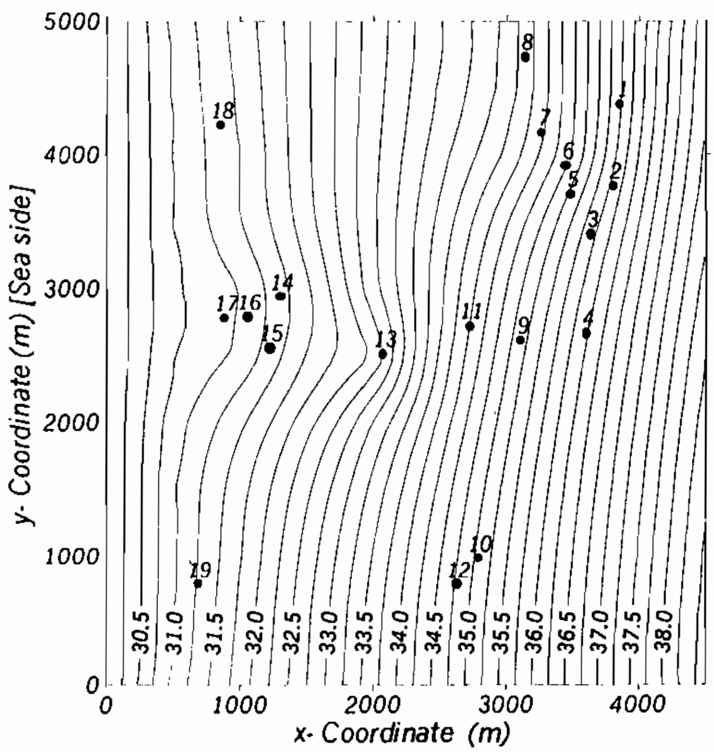

Figure 9 Water table level and well system, shaded area represents intruded area-zone II.

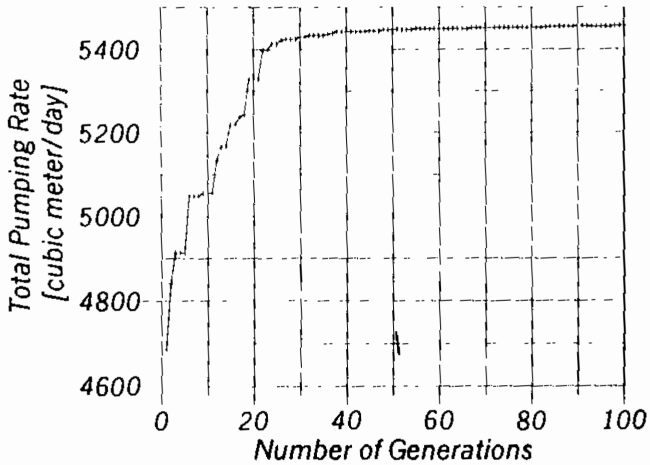

Figure 10 Optimal total pumping rate against number of generations.

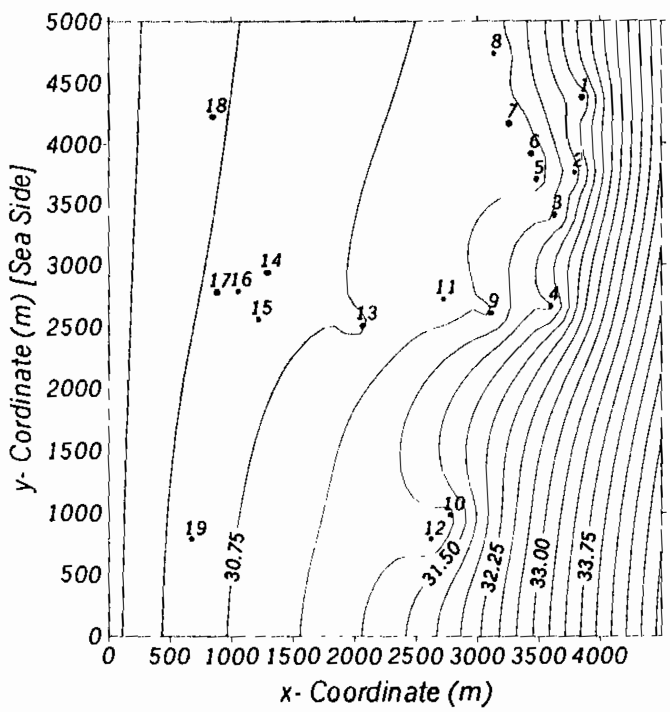

Figure 11 Water table level and well system, shaded area represents intruded area-zone II.