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NODAL LINE FINITE DIFFERENCE METHOD IN THE ANALYSIS OF RECTANGULAR PLATES ON ELASTIC FOUNDATION

طريقة الفروق المحددة للخطوط لتحليل الألواح المستطيلة المرتكزة على تربة مرنة

BY

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الخلاصة ـ يتناول هذا البحث تحليل البلاطات المستطيلة ذات الحواف الحرة والمرتكزة مباشرة على تربة مرنة elastic foundation وذلك باستخدام طريقة الفروق المحددة لخطوط التقسيم التي ابتكرها الباحث وسماها باسم nodal line finite difference method . وفي هذا التحليل تم تمثيل سلوك التربة المعقد رياضيا تحت هذه البلاطات بطريقة مبسطة تبعا لفرض فنكلر winkler assumption الذي يعتبر أن رد فعل التربة يتناسب طرديا مع الازاحة الناتجة من تأثير الأحمال الخارجية المرتكزة على هذه البلاطات . ولقد تم استخدام دالة أساسية basic function للتعبير عن الازاحة عند خطوط التقسيم بحيث تفي مسبقا بأحد شرطي الحواف الحرة عند طرفي هذه الخطوط . ولتحقيق الشرط الآخر تم التأثير بقوى عزوم عند أطراف خطوط التقسيم مساوية في المقدار ومخالفة في الاشارة لتلك العزوم الناتجة من استخدام الدالة الأساسية . ولقد تم تطبيق طريقة الباحث في تحليل نماذج من البلاطات المربعة والمستطيلة سبق حلها بطرق عددية وأظهرت النتائج تطابقا جيدا يدل على كفاءة هذه الطريقة ويشجع على استمرار استخدامها لحل مشاكل أخرى في هذا المجال .

ABSTRACT: *A nodal line finite difference bending analysis of isotropic rectangular plates with free lateral boundary conditions, using the nodal line finite difference method, is presented. The analysis describes the linear elastic behaviour of rectangular plates resting on a Winkler type foundation and loaded on its upper surface with arbitrary transverse loads. A basic function fits one of the boundary conditions of two opposite free ends is used to express the displacement variation along the nodal lines. To satisfy the other condition of the two opposite free ends, edge moments equal in magnitude but opposite in direction were applied at the ends of the nodal lines. Numerical results were obtained and compared with those obtained from another numerical solution. The comparison demonstrated a good agreement and indicated the validity of the presented technique.*

INTRODUCTION

The continuing and intensive interest for the improvement of the solution techniques used in the analysis of two and three dimensional problems has prompted the development of new semi-analytical methods among which the nodal line finite difference method NLFDM is one. The application of this method in the analysis of rectangular plates requires the division of the plate into a mesh of parallel fictitious nodal lines in one direction. The nodal line finite difference method calls for the use basic functions to express the displacement variation along these nc lines, with the stipulation that such functions should satisfy a priori boundary conditions at the ends of the nodal lines. Thus, the p differential equation is reduced to an ordinary differential equatior can be transformed into a nodal line finite difference equation central finite difference technique. The NLFDM method is similar finite strip method FSM developed by CHEUNG [1,2,3], since both c basic functions at nodal lines. The most commonly used basic f the eigen functions derived from the solution of beam vibratio equation. These basic functions have been worked out explici' [4] for different end conditions.

The nodal line finite difference method NLFDM was first introduced by the Author [7], using the trigonometric series as a basic function in the analysis of rectangular plates with two opposite simply supported ends. A basic function other than trigonometric series, was used by the Author [8] to analyze elastic rectangular plates with two opposite clamped ends. In this analysis, an iterative procedure was developed to overcome the coupling property of the static equilibrium equations. This iterative procedure is similar in concept to that developed earlier by the Author [5,6] for the bending analysis of rectangular plates by the finite strip method. The nodal line finite difference method has also been extended by the Author [9,10] to include the bending analysis of rectangular plates with variable flexural rigidity as well as with abrupt change in thickness in one direction.

The objective of the present work is to develop a nodal line finite difference solution for the analysis of rectangular plates on elastic foundation. The direct applications of this type of plates are for instance reinforced concrete pavement of highways and runways as well as the foundation rafts of buildings. The soil behaviour under such plates is of a non-linear nature, therefore it is quite difficult to be modelled since the deformation of the soil is not only a function of load intensity but also a function of time and rate of loading. To simplify the inherently complex problem, it is assumed that the supporting medium is isotropic, homogenous and linearly elastic. Such a type of subbase is called a Winkler type foundation. This assumption is not accurate enough to represent the actual soil behaviour, but in many cases it approximates closely the real situation. In the present work, elastic isotropic rectangular plates resting on elastic foundations are analyzed for free boundary conditions. A simple basic function in a form of cosine series was used to express the displacement variation along the nodal lines. The used basic function only satisfied the free boundary conditions with respect to the sheering forces, but resulted in bending forces at the ends of the nodal lines. In order to completely satisfy the free boundary conditions at the ends of the nodal lines, edge moments; equal in magnitude and opposite in direction to the resulted bending forces, have been applied and included in the analysis through the solution of the homogenous differential equation of the plate. The obtained results were compared with those obtained by BOWLES [11] and the comparison demonstrated a close agreement and indicated the validity of the presented technique.

METHOD OF ANALYSIS

1- Solution of the non-homogenous differential equation

a) Nodal Line Finite Difference Equation

According to the Winkler assumption, the subgrade reaction intensity is proportional to the deflection of the plate W. The intensity is then given by the expression $k_s W$, where the constant k_s, expressed in the term of stress per unit length of deflection, is called the modulus of the foundation or the subgrade reaction. In accordance with the Winkler assumption, the differential equation of the deflection of elastic isotropic rectangular plates becomes

$$B\,(W^{\cdot\cdot\cdot\cdot} + 2\,W^{\cdot\cdot\prime\prime} + W^{\prime\prime\prime\prime}) = q - k_s\,W \qquad \text{i.e}$$

$$B\,(W^{\cdot\cdot\cdot\cdot} + 2\,W^{\cdot\cdot\prime\prime} + W^{\prime\prime\prime\prime} + \rho\,W) = q \qquad (1)$$

where $(\ )^{\cdot} = \frac{\partial}{\partial x}$, $(\ )' = \frac{\partial}{\partial y}$,

$\rho = \frac{K_s}{B}$, K_s is the subgrade reaction and

$B = \frac{E\,t^3}{12(1-\nu^2)}$ is the flexural rigidity of the plate

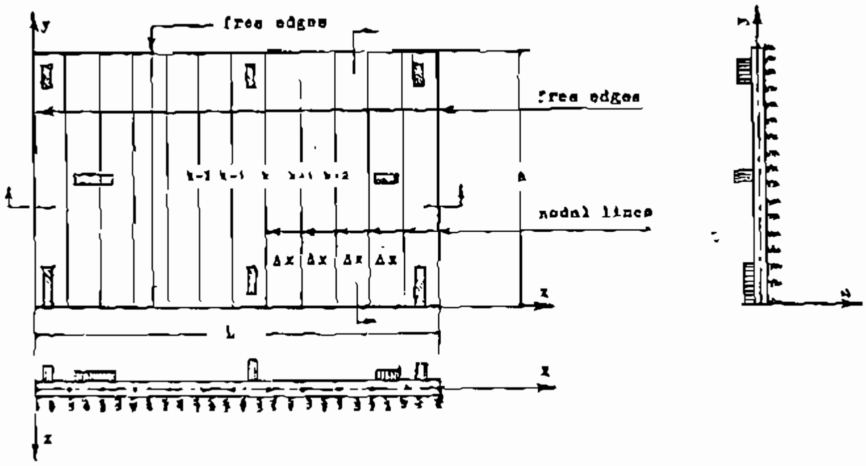

Fig. 1

In the application of the nodal line finite difference method for the analysis, the plate is divided into a mesh of fictitious nodal lines as shown in Fig. 1. The displacement function at each nodal line of the mesh is expressed as a summation of terms of the basic function fitting one of the two boundary conditions at the ends of the nodal lines multiplied by nodal line parameters. These parameters are assumed as single variable functions in the direction perpendicular to the nodal lines. The displacement function at any nodal line labelled k may be written as

$$W_k = \sum_{m=1}^{r} F_{m,k}(x)\ Y_m(y) \tag{2}$$

For rectangular plates with two opposite free ends, the basic function satisfying the boundary conditions with respect to shearing forces at the ends of the nodal lines is a series in the form

$$Y_m = \cos \frac{(m-1)\pi}{a}\, y = \cos k_m y \tag{3}$$

Resolving the load into a series similar to the used basic function and substituting equations (2) and (3) into equation (1) at any nodal line k leads to

$$B \sum_{m=1}^{r} [\, F_{m,k}^{\cdots\cdot} - 2k_m^2\, F_{m,k}^{\cdot\cdot} + (k_m^4+\rho)F_{m,k} \,]\ Y_m = \sum_{m=1}^{r} q_{m,k}\ Y_m \tag{4}$$

For each term of the basic function, equation (4) may be written as

$$B\ [\ F_{m,k}^{\cdots\cdot} - 2k_m^2\, F_{m,k}^{\cdot\cdot} + (k_m^4+\rho)F_{m,k} \,] = q_{m,k} \tag{5}$$

By applying the central finite difference technique, equation (5) can be written in a matrix form as follows

$$[1 \quad C^1_m \quad C^2_m \quad C^1_m \quad 1]\{F_{m,k-2}F_{m,k-1}F_{m,k} \quad F_{m,k+1}F_{m,k+2}\}^T = \frac{\Delta\bar{x}^4}{B} q_{m,k} \tag{6}$$

where $C^1_m = -(4+2\psi^2_m)$ and $C^2_m = (6+4\psi^2_m+\psi^4_m+\rho\Delta\bar{x}^4)$

Equation (6) represents the central nodal line finite difference equation for the different terms of the basic function

b) Internal Forces

For an elastic isotropic plates, the internal forces per unit length at any point are given by

$$\left.\begin{aligned}
M_x &= -B\,(W^{\cdot\cdot} + \nu W'') \\
M_y &= -B\,(W'' + \nu W^{\cdot\cdot}) \\
M_{xy} &= -M_{yx} = B\,(1-\nu)\,W^{\cdot\prime} \\
Q_x &= -B\,(W^{\cdot\cdot\cdot} + W^{\cdot\prime\prime}) \\
Q_y &= -B\,(W''' + W^{\cdot\cdot\prime}) \\
\bar{Q}_x &= -B\,[W^{\cdot\cdot\cdot} + (2-\nu)\,W^{\cdot\prime\prime}] = (Q_x - \frac{\partial M_{xy}}{\partial y}) \\
\bar{Q}_y &= -B\,[W''' + (2-\nu)\,W^{\cdot\cdot\prime}] = (Q_y - \frac{\partial M_{xy}}{\partial x})
\end{aligned}\right\} \tag{7}$$

By applying the central nodal line finite difference technique, the internal forces at any nodal line k may be written as

$$\left.\begin{aligned}
M_{x,k} &= -\frac{B\lambda^2}{a^2}\sum_{m=1}^{r} \cos k_m y\,[0 \quad 1 \quad -C^e_m \quad 1 \quad 0]\{\delta_m\} \\
M_{y,k} &= -\frac{B\lambda^2}{a^2}\sum_{m=1}^{r} \cos k_m y\,[0 \quad \nu \quad -C^4_m \quad \nu \quad 0]\{\delta_m\} \\
M_{xy,k} &= \frac{B\lambda^2}{2a^2}(1-\nu)\sum_{m=1}^{r} \psi_m \sin k_m y\,[0 \quad 1 \quad 0 \quad -1 \quad 0]\{\delta_m\} \\
Q_{x,k} &= -\frac{B\lambda^3}{2a^3}\sum_{m=1}^{r} \cos k_m y\,[-1 \quad C^5_m \quad 0 \quad -C^5_m \quad 1]\{\delta_m\} \\
Q_{y,k} &= -\frac{B\lambda^3}{a^3}\sum_{m=1}^{r} \psi_m \sin k_m y\,[0 \quad -1 \quad C^5_m \quad -1 \quad 0]\{\delta_m\} \\
\bar{Q}_{x,k} &= -\frac{B\lambda^3}{2a^3}\sum_{m=1}^{r} \cos k_m y\,[-1 \quad C^d_m \quad 0 \quad -C^d_m \quad 1]\{\delta_m\} \\
\bar{Q}_{y,k} &= -\frac{B\lambda^3}{a^3}\sum_{m=1}^{r} \psi_m \sin k_m y\,[0 \quad -1 \quad C^d_m \quad -1 \quad 0]\{\delta_m\}
\end{aligned}\right\} \tag{8}$$

where $C^e_m = (2+\nu\psi^2_m)$, $C^4_m = (2\nu+\psi^2_m)$, $C^5_m = (2+\psi^2_m)$, $C^d_m = \{2+(2-\nu)\psi^2_m\}$

$\{\delta_m\} = \{F_{m,k-2}F_{m,k-1}F_{m,k} \quad F_{m,k+1}F_{m,k+2}\}^T$

c Boundary Conditions

The NLFDM method requires the application of the nodal line difference equation at each nodal line of the plate including the edge nodal lines. Each edge nodal line difference equation will introduce two additional imaginary nodal lines outside the plate as shown in Fig. 2. According to the prescribed boundary conditions at the edge nodal lines, the parameters of the additional nodal lines have to be expressed in terms of the edge and the two adjacent interior nodal lines. The boundary conditions of free edge would be as

$$M_{x,k} = \bar{Q}_{x,k} = 0 \quad \text{i.e.} \quad (W^{''} + \nu W'')_k = 0 \;, \; \{ W^{'''} + (2-\nu) W^{'''} \}_k = 0 \qquad (9)$$

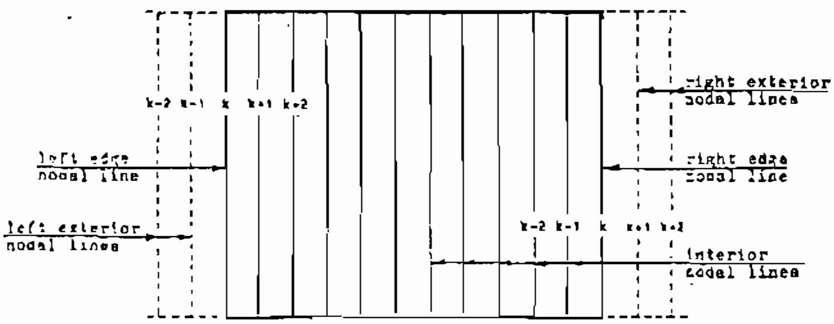

Fig. 2

Upon application of the central finite difference technique, left and right exterior nodal line parameters can be described according to the following relationships

$$\left.\begin{aligned} F_{m,k-1} &= C_m^s F_{m,k} - F_{m,k+1} \\ F_{m,k-2} &= C_m^s C_m^d F_{m,k} - 2C_m^d F_{m,k+1} + F_{m,k+2} \\ F_{m,k+1} &= C_m^s F_{m,k} - F_{m,k-1} \\ F_{m,k+2} &= C_m^s C_m^d F_{m,k} - 2C_m^d F_{m,k-1} + F_{m,k-2} \end{aligned}\right\} \qquad (10)$$

2- Solution of the homogenous differential equation

The used basic function only satisfied the free boundary conditions with respect to shearing forces but resulted in bending forces at ends of the nodal lines. The resulted bending forces at the ends of the nodal lines (y=0, y=a) are single variable functions in x direction.

$$\left.\begin{aligned} f(x)|_{y=0} &= M_1(x) + M_2(x) \\ f(x)|_{y=a} &= M_1(x) - M_2(x) \end{aligned}\right\} \qquad (11)$$

where $M_1(x)$ and $M_2(x)$ are two functions include ordinates of the bending forces resulted from the even and odd terms of the used basic function respectively.

Cosine series was choosen to express the variation of the resulted bending forces in x direction.

$$\left.\begin{aligned} M_1(x) &= \sum_{n=1}^{r} p_{1n} \cos \frac{(n-1)\pi}{L} x = \sum_{n=1}^{r} p_{1n} \cos \mu_n x \\ M_2(x) &= \sum_{n=1}^{r} p_{2n} \cos \frac{(n-1)\pi}{L} x = \sum_{n=1}^{r} p_{2n} \cos \mu_n x \end{aligned}\right\} \tag{12}$$

The coefficients p_{1n} and p_{2n} would be determined by numerical integration techniques.

In accordance with the Winkler assumption, the homogenoue partial differential equation of elastic isotropic plates takes the form

$$W'''' + 2\, W''{}'' + W'''' + \rho\, W = 0 \tag{13}$$

The solution of this equation may be expressed as

$$W = \sum_{n=1}^{r} \cos \frac{(n-1)\pi}{L} x \; {}^{*}Y_n = \sum_{n=1}^{r} \cos \mu_n x \; {}^{*}Y_n \tag{14}$$

Substitution of equation (13) into equation (14) leads to the following homogenous ordinary differential equation

$${}^{*}Y_n'''' - 2\,\mu_n^2 \; {}^{*}Y_n'' + (\mu_n^4 + \lambda^4)\, {}^{*}Y_n = 0 \tag{15}$$

$$\text{where} \quad \lambda^4 = \rho = \frac{k_s}{B}$$

General solution of this equation can be written in the following form

$${}^{*}Y_n = A_n\, Y_{1n} + B_n\, Y_{2n} + C_n\, Y_{3n} + D_n\, Y_{4n} \tag{16}$$

$$\begin{aligned} \text{where} \quad Y_{1n} &= e^{-\beta_n y} \cos \gamma_n y + e^{-\beta_n \bar{y}} \cos \gamma_n \bar{y} \,, \\ Y_{2n} &= e^{-\beta_n y} \sin \gamma_n y + e^{-\beta_n \bar{y}} \sin \gamma_n \bar{y} \,, \\ Y_{3n} &= e^{-\beta_n y} \cos \gamma_n y - e^{-\beta_n \bar{y}} \cos \gamma_n \bar{y} \,, \\ Y_{4n} &= e^{-\beta_n y} \sin \gamma_n y - e^{-\beta_n \bar{y}} \sin \gamma_n \bar{y} \,, \\ 2\beta_n^2 &= \sqrt{\mu_n^4 + \lambda^4} + \mu_n^2 \,, \quad 2\gamma_n^2 = \sqrt{\mu_n^4 + \lambda^4} - \mu_n^2 \quad \text{and} \quad \bar{y} = a - y \end{aligned}$$

For symmetry in y direction it is clear that ${}^{*}Y_n$ is an even function of y

$${}^{*}Y_n = A_n\, Y_{1n} + B_n\, Y_{2n} \tag{17}$$

For anti-symmetry in y direction it may be concluded that ${}^{*}Y_n$ is an odd function of y.

$${}^{*}Y_n = C_n\, Y_{3n} + D_n\, Y_{4n} \tag{18}$$

To satisfy the free boundary conditions for both shearing and bending forces at ends of the nodal lines, edge moments equal in magnitude to $M_1(x)$ and $M_2(x)$ but opposite in direction were applied. The constants A_n, B_n, C_n and D_n should be determined for each term of the function $\cos \mu_m x$ from the boundary conditions at $y=0$. For the edge bending forces resulted from the even terms of the used basic function, we have

$$\left.\begin{aligned} M_y\big|_{y=0} &= -B\left[W'' + \nu W^{\cdot\cdot}\right]_{y=0} = -M_1(x) \\ &= -B\sum_{n=1}^{r}\left[{}^{*}Y_n'' - \nu\mu_n^{2*}Y_n\right]_{y=0}\cos\mu_n x = -\sum_{n=1}^{r} P_{1n}\cos\mu_n x \\ \bar{Q}_y\big|_{y=0} &= -B\left[W''' + (2-\nu)W^{\cdot\cdot\prime}\right]_{y=0} = 0 \\ &= -B\sum_{n=1}^{r}\left[{}^{*}Y_n''' - (2-\nu)\mu_n^{2*}Y_n'\right]_{y=0}\cos\mu_n x = 0 \end{aligned}\right\} \quad (19)$$

Substituting equation (17) into equation (19) gives for each term of the function $\cos \mu_m x$ the following relations

$$\left.\begin{aligned} A_n\left[Y_{1n}'' - \nu\mu_n^2 Y_{1n}\right]_{y=0} + B_n\left[Y_{2n}'' - \nu\mu_n^2 Y_{2n}\right]_{y=0} &= \frac{P_{1n}}{B} \\ A_n\left[Y_{1n}''' - (2-\nu)\mu_n^2 Y_{1n}'\right]_{y=0} + B_n\left[Y_{2n}''' - (2-\nu)\mu_n^2 Y_{2n}'\right]_{y=0} &= 0 \end{aligned}\right\} \quad (20)$$

The same steps were applied to the edge bending forces resulted from the odd terms of the basic function, obtaining

$$\left.\begin{aligned} C_n\left[Y_{3n}'' - \nu\mu_n^2 Y_{3n}\right]_{y=0} + D_n\left[Y_{4n}'' - \nu\mu_n^2 Y_{4n}\right]_{y=0} &= \frac{P_{2n}}{B} \\ C_n\left[Y_{3n}''' - (2-\nu)\mu_n^2 Y_{3n}'\right]_{y=0} + D_n\left[Y_{4n}''' - (2-\nu)\mu_n^2 Y_{4n}'\right]_{y=0} &= 0 \end{aligned}\right\} \quad (21)$$

The constants A_n, B_n, C_n and D_n may expressed as

$$\left.\begin{aligned} A_n &= \frac{a_{4n}}{a_{1n}a_{4n} - a_{2n}a_{3n}}\,\frac{P_{1n}}{B}, \qquad & B_n &= -\frac{a_{3n}}{a_{4n}}A_n \\ C_n &= \frac{b_{4n}}{b_{1n}b_{4n} - b_{2n}b_{3n}}\,\frac{P_{2n}}{B}, \qquad & D_n &= -\frac{b_{3n}}{b_{4n}}C_n \end{aligned}\right\} \quad (22)$$

where
$$a_{1n} = \varepsilon_n^2(1+\psi_1) + \lambda^2\psi_2, \qquad a_{2n} = \varepsilon_n^2\psi_2 - \lambda^2(1+\psi_1),$$
$$a_{3n} = \varepsilon_n^2\left\{\beta_n(1-\psi_1) - \gamma_n\psi_2\right\} + \lambda^2\left\{\beta_n\psi_2 + \gamma_n(1-\psi_1)\right\},$$
$$a_{4n} = -\varepsilon_n^2\left\{\beta_n\psi_2 + \gamma_n(1-\psi_1)\right\} + \lambda^2\left\{\beta_n(1-\psi_1) - \gamma_n\psi_2\right\},$$
$$b_{1n} = \varepsilon_n^2(1-\psi_1) - \lambda^2\psi_2, \qquad b_{2n} = \varepsilon_n^2\psi_2 - \lambda^2(1-\psi_1),$$
$$b_{3n} = \varepsilon_n^2\left\{\beta_n(1+\psi_1) + \gamma_n\psi_2\right\} - \lambda^2\left\{\beta_n\psi_2 - \gamma_n(1+\psi_1)\right\},$$
$$b_{4n} = \varepsilon_n^2\left\{\beta_n\psi_2 - \gamma_n(1+\psi_1)\right\} + \lambda^2\left\{\beta_n(1+\psi_1) + \gamma_n\psi_2\right\},$$
$$\varepsilon_n^2 = (1-\nu)\mu_n^2, \quad \psi_1 = e^{-\beta_n a}\cos\gamma_n a \ \text{ and } \ \psi_2 = e^{-\beta_n a}\sin\gamma_n a$$

Finally, deflection and internal forces can be calculated and added to the solution of the non-homogenous differential equation.

NUMERICAL EXAMPLES

To demonstrate the validity of the proposed solution technique, analysis of rectangular plates on elastic foundation was carried out. For the purpose of comparison, two problems solved previously by BOWLES [11] were chosen.

Example 1: A problem of rectangular spread footing subjected to central column load shown in Fig. 3 was analyzed. Due to symmetry in x direction, only half of the plate divided into a mesh of fictitious nodal lines at equal distance(Δx =0.3 ms) was considered. The analysis was carried out using seven even terms of the used basic function. To illustrate the effect of the applied load area, different column dimensions were taken into consideration. The results of deflection, moments M_x and M_y at selected nodes on the central line of symmetry (y=0.9 ms) and the free edge (y=0) were presented in tables 1 and 2. Comparison of the results of the proposed solution technique with those obtained by BOWLES demonstrated a significant effect of the applied load area, especially on the value of the moments M_x and M_y at the central point. It should be noted that the edge moment at the free edge (y=0) is nearly equal to zero. this indicates the power of the proposed solution technique for satisfying the free boundary conditions. The data of the problem was taken from BOWLES [11] (example 7-3 page 222) as follows

Modulus of elasticity
E = 2240873 kN/sqm
= 228.49729 t/cm2

Subgrade reaction
k_s = 23536 kN/cum
= 2.3999184 kg/cm3

Column load
P = 890 kN
= 90.751504 ton

Poisson's ratio
ν = .15

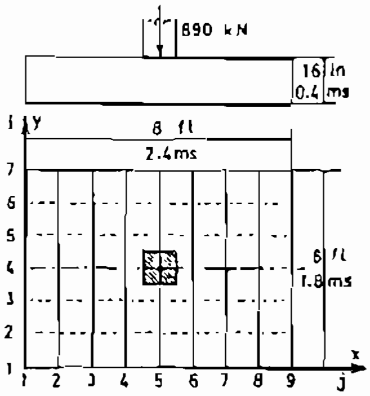

Fig. 3

Table 1. Deflection w, Bending Moments Mx and My at y=0.9 ms.

	column dimensions cm	i j 4 5	i j 4 6	i j 4 7	i j 4 8	i j 4 9		SOURCE
w	30x30	0.9420	0.9357	0.9232	0.9089	0.8944	cm	NLFDM
	25x25	0.9424	0.9358	0.9233	0.9088	0.8942	cm	
	20x20	0.9428	0.9360	0.9233	0.9087	0.8939	cm	
	10x10	0.9436	0.9362	0.9233	0.9084	0.8935	cm	
	point load	0.9443	0.9365	0.9232	0.9082	0.8930	cm	
	point load	0.9453	0.9371	0.9238	0.9090	0.8946	cm	BOWLES[11]
Mx	30x30	19.491	9.746	3.474	0.723	0.221	t.m	NLFDM
	25x25	20.417	9.614	3.440	0.715	0.222	t.m	
	20x20	21.344	9.463	3.407	0.700	0.222	t.m	
	10x10	23.132	9.116	3.348	0.698	0.221	t.m	
	point load	24.696	8.748	3.302	0.692	0.218	t.m	
	point load	25.245	8.640	2.859	0.306	0.000	t.m	BOWLES(11)
		247.503	84.814	20.042	3.003	0.000	kN.m	
My	30x30	15.840	9.910	5.563	3.605	2.922	t.m	NLFDM
	25x25	17.113	10.106	5.600	3.620	2.933	t.m	
	20x20	18.393	10.236	5.616	3.623	2.934	t.m	
	10x10	20.714	10.204	5.591	3.602	2.914	t.m	
	point load	22.269	10.070	5.506	3.553	2.876	t.m	
	point load	20.554	11.445	6.417	4.104	3.300	t.m	BOWLES[11]
		202.551	112.243	62.931	40.253	32.364	kN.m	

Table 2 Deflection w, Bending Moments Mx and My at y=0.0 ms.

	column dimensions cm	i j i 5	i j i 6	i j i 7	i j i 8	i j i 9		SOURCE
w	30x30 25x25 20x20 10x10 point load	0.9256 0.9257 0.9258 0.9261 0.9264	0.9221 0.9222 0.9222 0.9224 0.9227	0.9130 0.9130 0.9130 0.9130 0.9130	0.9007 0.9006 0.9005 0.9003 0.9001	0.8871 0.8869 0.8867 0.8862 0.8858	cm cm cm cm cm	NLFDM
	point load	0.9247	0.9211	0.9120	0.8997	0.8864	cm	BOWLES(11)
Mx	30x30 25x25 20x20 10x10 point load	9.319 9.433 9.556 9.822 10.008	7.506 7.642 7.697 7.808 7.920	4.391 4.414 4.435 4.475 4.516	1.795 1.805 1.813 1.823 1.831	0.563 0.568 0.570 0.572 0.573	t.m t.m t.m t.m t.m	NLFD
	point load	9.637 94.509	7.650 75.106	4.339 42.549	1.657 16.249	0.000 0.000	t.m kN.m	BOWLES(11)
My	30x30 25x25 20x20 10x10 point load	-0.006 -0.005 -0.004 0.000 0.003	-0.023 -0.021 -0.018 -0.008 -0.002	-0.012 -0.012 -0.011 -0.009 -0.000	0.007 0.007 0.007 0.006 0.006	0.063 0.062 0.061 0.060 0.059	t.m t.m t.m t.m t.m	NLFDM
	point load	0.000 0.000	0.000 0.000	0.000 0.000	0.000 0.000	0.000 0.000	t.m kN.m	BOWLES(11)

EXAMPLE 2: A problem of rectangular raft foundation subjected to 12 column loads shown in Fig. 4 was analyzed. The raft was divided into a mesh of fictitious nodal lines in x direction at equal distance (Δx=3.7 ft, 21 nodal lines). The analysis was carried out using fourteen even and odd terms of the basic function. Using the proposed solution technique, a final square matrix having a band width equal to 5 stored in a rectangular matrix with the dimension 21x5 was solved. At first, the problem was solved by considering a patch column loads (15x15 in) and secondly by assuming a point column loads. The results of the moments Mx and My at selected nodes on the nodal lines 4, 8 and 11 were presented in tables 3, 4 and 5. The results are presented using the same unites and sign convention considerad by BOWLES (+ sign of moment indicates tension at the upper surface of the raft). BOWLES considered the column loads as point loads and divided the raft into a mesh of 21x15 nodes. AS a results, a final fully populated square matrix with the dimension 315x315 was solved. Comparison of the obtained results with those obtained by BOWLES shows a good agreement. The data of the problem was taken from BOWLES[11] (example 7-2, page 219) as follows

All column dimensions are 15x15 in

Modulus of elasticity
E = 468000 Ksf

Subgrade reaction
k_s = 36 Kcf

Raft thickness
t = 3.833 ft

Poisson's ratio
ν = .15

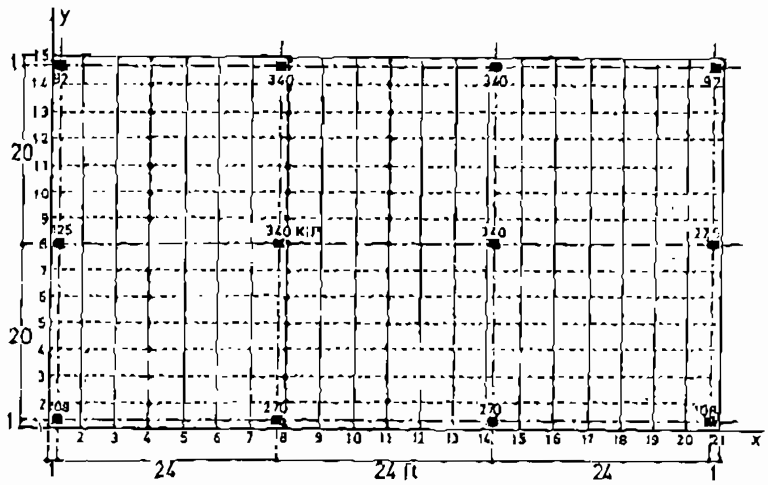

Fig. 4

Table 3. Bending Moments Mx and My at nodal line No 4

Point	Distance y ft	Moment Mx kip.ft/ft			Moment My kip.ft/ft		
		patch load	point load	point load	patch load	point load	point load
15	42	53.633	47.490	50.206	- 0.006	- 0.006	0.000
14	39	46.147	40.013	40.033	10.119	18.079	17.820
13	36	39.951	33.813	34.042	34.072	34.017	33.780
12	33	35.764	29.630	30.009	45.631	45.543	45.280
11	30	33.824	27.701	20.272	51.617	51.483	51.160
10	27	33.849	27.738	28.540	52.400	52.213	51.670
9	24	34.958	28.863	30.140	49.999	49.759	48.890
8	21	35.551	29.467	31.160	47.935	47.664	45.940
7	18	34.531	28.435	29.930	48.515	48.268	47.250
6	15	32.946	26.833	28.730	49.648	49.448	49.000
5	12	32.346	26.218	27.890	48.021	47.869	47.640
4	9	33.568	27.437	29.240	41.786	41.600	41.530
3	6	36.853	30.725	32.640	30.690	30.620	30.510
2	3	42.049	35.901	37.000	16.001	15.952	15.910
1	0	48.441	42.279	46.700	0.000	- 0.001	0.000
SOURCE		NLFDM		BOWLES(11)	NLFDM		BOWLES(11)

Table 4. Bending Moments Mx and My at nodal line No 8

Point	Distance y ft	Moment Mx kip.ft/ft			Moment My kip.ft/ft		
		patch load	point load	point load	patch load	point load	point load
15	42	-120.846	-126.280	-117.610	2.114	2.149	0.000
14	39	- 78.784	- 83.435	- 80.170	15.849	16.564	18.450
13	36	- 44.625	- 49.106	- 47.720	52.181	52.571	50.200
12	33	- 27.409	- 31.980	- 30 400	64.137	64.230	63.810
11	30	- 20.952	- 25.442	- 23 840	60.220	60.224	67.730
10	27	- 23 339	- 27.020	- 26 050	63 523	63.406	62.600
9	24	- 35 777	- 40.273	- 37.230	41.543	41.657	44.580
8	21	- 49 460	- 54.297	- 57 240	4.048	3.095	5.410
7	18	- 34.113	- 38.612	- 35.520	39.246	39.345	42.320
6	15	- 19 770	- 24.258	- 22 270	59.232	59.166	58.530
5	12	- 15 005	- 19 500	- 17 630	62.306	62.272	61.980
4	9	- 18.120	- 22.643	- 20 800	57 677	57 705	52.510
3	6	- 30.250	- 34.756	- 33.000	46 087	46.358	44.590
2	3	- 56.152	- 60.000	- 58.320	15.017	15.561	17.160
1	0	- 88.523	- 93.804	- 85 650	1.682	1.709	0.000
SOURCE		NLFDM		BOWLES(11)	NLFDM		BOWLES(11)

Table 5. Bending Moments Mx and My at nodal line No 11

Point	Distance y ft	Moment Mx kip.ft/ft			Moment My kip.ft/ft		
		patch load	point load	point load	patch load	point load	point load
15	42	28.946	29.110	29.210	- 0.695	- 0.706	0.000
14	39	21.430	21.506	19.010	19.098	19.302	18.860
13	36	15.902	15.929	13.580	37.063	37.405	36.910
12	33	13.012	13.026	10.740	50.256	50.650	50.100
11	30	12.882	12.903	10.630	56.552	56.946	56.300
10	27	14.935	14.977	12.500	56.300	56.664	55.700
9	24	17.935	17.908	15.030	52.324	52.639	51.090
8	21	19.753	19.819	18.110	49.380	49.660	47.220
7	18	19.023	19.077	16.950	50.419	50.716	49.230
6	15	17.041	17.077	14.770	52.032	53.161	52.400
5	12	15.010	15 035	13 560	52.181	52.527	52.040
4	9	16.463	16.404	14.380	45.890	46.227	45.870
3	6	19.450	19.481	17 330	33.657	33.942	33.720
2	3	24.635	24.708	22.430	17.319	17.488	17.110
1	0	31.469	31.614	31.730	- 0.547	- 0.556	0.000
SOURCE		NLFDM		BOWLES(11)	NLFDM		BOWLES(11)

CONCLUSION

In this investigation, analysis of rectangular plates with free boundary conditions supported on elastic foundation was achieved by using the nodal line finite difference method. A simple trigonometric basic function in the form of cosine series was used to express the displacement variation along the nodal lines. The used basic function has the advantage of uncoupled system of the static equilibrium equations. The basic function has the property to satisfy the free boundary conditions with respect to shearing forces, but resulted in bending forces at the ends of the nodal lines. In order to satisfy the free boundary conditions with respect to the bending forces at the ends of the nodal lines, edge moments equal in magnitude and opposite in direction to the resulted bending forces were applied. To determine the effects of the applied edge moments, it would be easy to solve the homogenous part of the differential equation of the plate. A comparison of the obtained results with those available from the finite difference solution of BOWELS shows a close agreement.

NOTATIONS

W = transverse deflection.

a = length of the nodal lines.

L, a = dimensions of the plate.

Δx = constant distance between the nodal lines.

E = modulus of elasticity.

t = thickness of the plate.

ν = possion's ratio.

B = flexural rigidity of the plate.

k_s = subgrade reaction of the soil.

$F_{m,k}$ = nodal line parameters.

Y_m = basic function.

q = load intensity.

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