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WAVEFORM ESTIMATION TECHNIQUES FOR EVENT-RELATED BIOELECTRIC SIGNALS: A COMPARATIVE STUDY OF PERFORMANCE

دراسة لمقارنه أداء تقنيات استخلاص الموجات للاشارات الحيويه

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الخلاصه :

هناك العديد من الأشارات الحيويه التى تنشأ من الإستجابه الكهربيه للمنظومه الفسيولوجيه لنبضه حثيه هذه النبضه تكون أما داخليه كما فى اشارات رسم القلب الكهربى أو خارجيه كما فى الاشارات العصبيه وفى هذا البحث نقدم دراسة مقارنه لاداء سبعه تقنيات تستخدم عاده فى استخلاص الموجات من الإشارات المرتبطه والمتزامنه مع حدوث مؤثر . وبإستخدام المحاكاة الرقميه لعدة إشارات وإضافة ضوضاء اليها بعدة نسب مختلفة أمكن الحصول على مجموعه من الموجات المحاكاه لإشارات حقيقيه . وقد إستخدمنا لمقارنة الأداء الجزر التربيعى لمتوسط مربعات الفروق ومقياسين لتقدير نسبة الإشاره الى الضوضاء قد أظهرت النتائج أن المرشح المتكيف ذا النبضه الترابطيه هو أكثا التقنيات لمقدرتة على تقدير المركبه المحدده للإشاره والتخلص من الضوضاء الغير مترابطه مع الإشاره .

ABSTRACT- Many bioelectric signals result from the electrical response of physiological system to an impulse that can be internal (ECG signals) or external (evoked potentials). A comparative study of performance of seven waveform estimation techniques used for event-related signals that are time-locked to a stimulus is presented in this paper. Computer generated signals and noise for several signal-to-noise ratios (SNR's) are used to make ensembles of simulated noisy waveforms. The performance of each technique is numerically investigated using the root-mean-squared error and two well known SNR estimators. The results show that an adaptive impulse correlated filter (AICF) performs the best. It is capable to estimate the deterministic component of the signal and removes the noise uncorrelated with stimulus even if this noise is coloured.

I. INTRODUCTION

Among the most well-studied bioelectrical signals are the event-related signals that are time-locked to a stimulus. The stimulus is usually external (visual, auditory, or electrical in the case of evoked potentials). In other cases the signal is related to an internal stimulus. In these cases a time-reference point can be defined from a wave of the same signal, as with QRS complex when analysing ECG signals.

Bioelectrical signals are often contaminated by noise from various sources. In general, an event-related signal can be considered as a process which can be decomposed into an invariant deterministic signal time-locked to a stimulus, and an additive noise uncorrelated with the signal. The most common signal processing of this type of bioelectric signal separates the deterministic signal from the noise. In recent years, a variety of techniques have been described. Linear filtering is not possible in general, because the spectra of signal and noise overlap. The conventional ensemble average (EA) technique [1] is an effective method for recovering the signal hidden in noise. However, it fails in analysis of nonstimulus-locked or variable latency signals unless latency variation is removed before averaging or time alignment of signals is first performed as in the coherent averaging (CA) technique [2-4].

One of the techniques which attempted for the variability of latencies was introduced in 1967 by Woody [5,6]. Although the method represents a significant step forward over ensemble and coherent averaging, it may still leave buried in the noise much of the information inherent in the signal, such as independent shifts in latency and amplitude in the components of the individual waveforms. Another technique was developed in which an evoked response is obtained by weighting each single waveform prior to averaging [7]. The technique is known as weighted averaging (WA) and the weights used must satisfy a generalized eigenvalue problem involving the correlation matrices of the underlying signal and noise components in order to maximize the signal-to-noise ratio (SNR) of the resulting average.

An adaptive impulse correlated filter (AICF) that can be applied to evoked potentials and low-amplitude potentials that are time-locked to high-amplitude wave of the ECG has been proposed by Laguna et al. [8]. The filter estimates the deterministic component of the signal and removes the noise uncorrelated with the stimulus even if this noise is colored [9]. A spectral averaging (SA) technique was developed [10-12]. It uses a scaled average of the unwrapped Fourier phases of the noisy signals and is shown to overcome latency variations.

Recently, Nakamura 1993 [13], has developed a technique that improves the signal-to-noise ratio of repetitive signals with variable delay. The technique is based on bispectral averaging (BA) and recovers the signal waveform from a set of noisy signals with variable latencies and does not require explicit time alignment of signals.

The purpose of the present study is to compare quantitatively the performance of seven different waveform estimation techniques. Five techniques, the ensemble averaging (EA), the coherent averaging (CA), the Woody's technique (WT), the weighted averaging technique (WA) and the adaptive impulse correlated filter technique (AICF) are time domain techniques and are independent of fluctuations in the baseline of the waveforms. The other two methods, the spectral averaging (SA) and the bispectral averaging (BA) are based on transformation techniques and the baseline movements must be removed before application. Comparison was performed by simulations of three different computer-generated signal sequences with uniform distribution signal delays and corrupted with white or coloured noise. Improvement of the SNR of the waveform estimate is examined numerically by way of the root-mean-squared error and two well-known SNR estimators. Advantages and disadvantages of the seven techniques are discussed.

II. THEORETICAL BACKGROUND

In this section, a description of each of the seven techniques is reported. Since the theoretical background of these techniques has been covered extensively in the literature [1-16], hence, only a brief summary of some points relevant to each method is given.

Consider an ensemble of M of noisy waveforms:

$$X(t) = [x_1(t) \quad x_2(t) \quad \ldots \quad x_M(t)\ ]^T \qquad (1)$$

for the time being, we will assume the ith waveform. is a continuous-time signal given by

$$x_i(t) = s_i(t) + n_i(t) \qquad i = 0, 1, \ldots, M \quad 0 \le t \le T \tag{2}$$

where $s_i(t)$ and $n_i(t)$ are the respective "signal" and "noise" components in the ith waveform. This model has been widely accepted in the literature. For the time being we make no further assumptions about $s_i(t)$ and $n_i(t)$. The ensemble of constituent signal and noise components are given by:

$$S(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_M(t) \end{bmatrix} \quad \text{and} \quad N(t) = \begin{bmatrix} n_1(t) \\ n_2(t) \\ \vdots \\ n_M(t) \end{bmatrix} \tag{3}$$

respectively, so that $X(t) = S(t) + N(t)$

A- Ensemble Averaging

The average of the ensemble of M waveforms can be used as an estimate of the signal

$$y(t) = \frac{1}{M} \sum_{i=1}^{M} x_i(t) \tag{4}$$

It can be shown [1] that if N(t) is a zero-mean stationary process, uncorrelated from waveform to another and uncorrelated with S(t), then the ensemble average forms a consistent signal estimator, i.e.

$$E\left\{ y(t) \right\} = S(t) \tag{5}$$

and

$$Var\left\{ y(t) \right\} = \frac{1}{M} \; Var\left\{ n(t) \right\} \tag{6}$$

where E(y(t)) is the expected value and Var(y(t)) is the variance of y at instant t. Hence, it may be concluded that the signal-to-noise ratio improves with a factor $M^{1/2}$.

The theory of ensemble averaging so far considered has assumed that the signal S(t) is invariant from waveform to waveform. If this were to be true, then averaging would probably be the optimum waveform estimation technique (so long as the average noise term N(t) tends to zero as the number of waveforms M increases). This, however, is not the case; in fact: there is considerable variability among an ensemble of bioelctric waveforms.

B- Coherent Averaging

Coherent averaging is a process whereby fixed intervals of a noisy signals are aligned temporally with respect to a reference point and then summed [2]. In the ideal case, M averages will reduce the noise by a factor of $1/M^{1/2}$ while leaving the signal uneffected (Eq.5). In practice, certain features of both noise and signal may limit the efficacy of coherent averaging. The two conditions for optimal averaging are: 1) the noise contaminent must be both random and stationary and 2) the signal of interest should be precisely synchronized to a stable reference time, or fiducial point. Variability in this time reference namely the time jitter, results in a distortion [2].

C- Woody's Cross-Correlation Method

In the Woody procedure [5], the individual waveforms are aligned to one another before averaging. The alignment is accomplished by finding the latency shift that gives a maximum cross-correlation coefficient between the waveform and a template formed by the averaging of the previously aligned waveforms. The individual waveforms are then corrected for their average latency variations and an average waveform computed. By repeating this with the average signal used as the template a further refinement in the latency estimates is obtained and a new average signal computed. This iterative procedure leads to an improved estimate of the shape of the signal when the only variable is the total signal latency.

D- Weighted Averaging

The weighted ensemble average of M waveforms can be written as

$$y(t) = W^T . X(t). \qquad (7)$$

where $W^T = [w_1, w_2, \ldots, w_M]$ is the M-by-1 weight vector. These weights are shown to maximize the signal-to-noise ratio (SNR) of the resulting average if they satisfy a generalized eigenvalue problem involving the correlation matrices of the underlying signal and noise components. The signal and noise correlation matrices are difficult to estimate and the solution of the generalized eigenvalue problem is often computationally impractical for real-time processing [7]. Correspondingly, a number of simplifying assumptions about the signal and noise correlation matrices were made which allow an efficient method of approximation. The method of optimizing the weights is described in detail in reference [7] and the optimal weights are given by:

$$W^* = \frac{\bar{X} X^T}{|| \bar{X} X^T ||} \qquad (8)$$

where division by the Euclidian norm, $||\bar{X}X^T||$ ensures that W^* has a norm of unity. The constraints imposed in deriving (8) allow the weight vector to be calculated with minimum amount of computational effort even for large values of M compared to what could be required to compute W.

E- Adaptive Impulse Correlated Filter

The objective of the adaptive impulse correlated filter (AICF) technique is to adapt filter coefficients or weights so that the impulse response of the desired waveform is acquired [8]. The filter

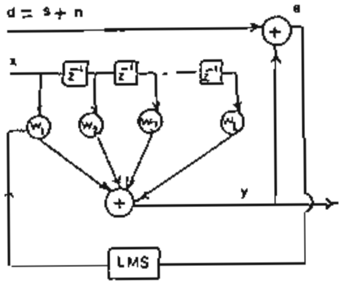

Fig.1 Block diagram of Adaptive Impulse Correlated Filter (AICF)

needs two inputs: the signal (primary input) and another input correlated with the deterministic component (reference input). The primary input (X_k) is the consecutive linking of the M recurrences of the event-related signal we want to filter: Each event related signal extends the interval of interest following the stimulus and is considered as a record of a random process. Let the waveform span k = 0 ... (J - 1) samples, and therefore the transversal filter will require L weights. The reference input D_k is a unit impulse synchronized with the beginning of each waveform x_k (the stimulus). Each recurrence i = 1, ..., M of the waveform results in a new reference impulse and a new update of all the filter weights (Fig.1). The output of this adaptive filter y_k can be expressed by

$$y_k = \sum_{i=1}^{L} w_{ik} \; d_{k-i} = W_k^T \; D_k \qquad (9)$$

where $W_k = [w_{1k} \;\; w_{2k} \;\; \ldots \;\; w_{Lk}]^T$ is the weight vector and $D_k = [0, 0, 1, \ldots, 0]^T$. The desired response is obtained by minimizing the mean square error between the primary and reference inputs. Therefore, the weights are given by

$$W_{k+1} = W_k + 2 \; \mu \; \epsilon_k \qquad (10)$$

where $\epsilon_k = d_k - y_k$ and μ is the convergence factor. At each time step only one filter weight is adapted. All the filter weights are adapted once each recurring waveform.

F- Spectral Averaging

A simple time delay manifests itself as an additive linear phase in the frequency domain. Spectral averaging circumvents the problem of variable latencies by averaging the phase harmonics in the frequency domain. Consider an ensemble of noise-free waveforms

$$r_i(t) = s_i(t - \tau_i) \qquad i = 1, 2, \ldots, M \qquad (11)$$

$$0 \le t \le T$$

where τ_i is the waveform time delay. The Fourier transform of (11) is

$$R_i(\omega) = |R_i(\omega)| \;\; \exp(j\phi_{r_i}(\omega)), \quad i = 1, \ldots, M \qquad (12)$$

where $|R_i(\omega)| = |s_i(\omega)|$

and $\phi_{r_i}(\omega) = \phi_{s_i}(\omega) - \omega\,\tau_i$

and where ϕ represents the Fourier phase. If we could now use these phases, $\phi_{r_i}(\omega)$, to compute $\phi_{s\bar{\tau}}(\omega)$, where

$$\phi_{s\bar{\tau}}(\omega) = \phi_s(\omega) - \omega\,\bar{\tau} \qquad (13)$$

and $$\bar{\tau} = \frac{1}{M}\sum_{i=1}^{M}\tau_i$$

then, the signal can be reconstructed at its mean delay.

$$s(t - \bar{\tau}) = F^{-1}\left\{ \,|\, S(\omega)\,|\, \exp(j\phi_{s\bar{\tau}}(\omega))\right\} \qquad (14)$$

where F^{-1} stands for the inverse Fourier transform of the quantity in brakets. We cannot simply average the principal values of the waveforms $\phi_{r_i}(\omega)$ to produce $\phi_{s\bar{\tau}}(\omega)$ since, in general, averaging principal values gives a biased estimate of the principal value of the average. Two different approaches are to be followed to implement the spectral averaging technique [11]: the unwrapped phase method and the phase vector technique.

G- Bispectral Averaging

Bispectral averaging is used to recover the signal waveforms from a set of observed noisy signals with variable delay. The bispectrum is the Fourier transform of the triple correlation [14]. It is related to the Fourier coefficients by

$$\tilde{X}^{(3)}(u,v) = \tilde{X}(u)\ \tilde{X}(v)\ \tilde{X}(-u - v) \qquad (15)$$

where $\tilde{X}(u)$ is the Fourier transform of $x(t)$,

$$\tilde{X}(u) = \int x(t)\exp(-2\pi jut)dt \qquad (16)$$

If the signal $x(t)$ is real, then

$$\tilde{X}(-u) = \tilde{X}^{*}(u) \qquad (17)$$

where * denotes complex conjugate.

The bispectrum can recover information about both amplitude and phase of the Fourier transform of the signal. Several procedures for the recovery of the Fourier amplitude and the Fourier phase from the bispectrum have been reported [15,16]. These can be roughly classified into two basic approaches; one is referred to as the recursive method and the other as the least squares method.

To perform the bispectral averaging, the first step is to compute the Fourier transform of the noisy signals. Before computing the Fourier transform, the mean values of the noisy signals must be removed. The bispectrum is computed using (15). Bispectra are then averaged for the ensemble of noisy signals. Fourier amplitudes and phases are recovered from the ensemble averaged bispectrum. Finally, signal recovery is performed by the inverse Fourier transform and the mean value is added. Recovering the amplitudes and phases is performed using the recursive method [16]. It is based on (15). The amplitude of the bispectrum is given by

$$\alpha(u,v) = A(u)A(v)A(-u-v) \tag{18}$$

where $\alpha(u,v)$ and $A(u)$ are the bispectrum amplitude and the Fourier amplitude, respectively. In discrete notation, Eq.(17) is written for real signal as

$$\alpha_{i,j} = A_i A_j A_{i+j} \tag{19}$$

where $\alpha_{i,j}$ and A_i represent the sampled bispectrum amplitude and the sampled Fourier amplitude of a signal, respectively. In (19), we used the fact that, for a real signal $x(t)$, $A_i = A_{-i}$. From (19), the Fourier amplitudes $A(k)$, $(k = 2, 3, \ldots, N)$ can be obtained recursively except the amplitude A_1, where $A_i (i = 1, 2, \ldots, N)$ are assumed to be nonzero. As proposed in the literature [16], A_1 can be calculated by

$$A_1 = \left[\frac{(\alpha_{1,1})^3 \; \alpha_{3,1}}{\alpha_{2,1} \; \alpha_{2,2}} \right]^{1/6} \tag{20}$$

A_0 can be determined by the sample mean [16].

Similarly, from (15) the phase of the bispectrum is given by

$$\beta(u,v) = \phi(u) + \phi(v) - \phi(u+v) \tag{21}$$

where $\beta(u,v)$ and $\phi(u)$ are the bispectrum phase and the Fourier phase, respectively. In discrete notation, (21) is written as

$$\beta_{i,j} = \phi_i + \phi_j - \phi_{i+j} \tag{22}$$

where $\beta_{i,j}$ and ϕ_i represent the sampled bispectrum phase and the sampled Fourier phase of a signal, respectively. In (21), we used the fact that for a real signal $x(t)$, $\phi_i = -\phi_{-i}$. By setting $i = 1$ in (22), we obtain the following equation for $j = 1, 2, \ldots, N-1$:

$$\phi_{1+j} = \phi_1 + \phi_j - \beta_{1,j} \tag{23}$$

N is the total number of Fourier phase unknowns. From (23), the Fourier phase ϕ_k $(k = 2, 3, \ldots, N)$ can be obtained recursively except for the phase ϕ_1. The phase ϕ_1 can be set arbitrary, for instance, $\phi_1 = 0$.

III. SIMULATIONS

Computer simulations have been conducted with three different signal sequences which are shown in Fig.2. The curves were generated using the following equations:

$$s_1(t) = \exp(-2t)\sin(4\pi t) \quad (24)$$

$$s_2(t) = (4t)\exp(-8t) \quad (25)$$

$$s_3(t) = [t(t-0.8)(t-2.4) + 0.1t]\exp[-(t-1.2)^2] \quad (26)$$

The curves consisting of 25 points were generated at 0.05 intervals. The mean values of $s_1(t)$, $s_2(t)$ and $s_3(t)$ are 0.085, 0.336, and 0.189, respectively. The maximum absolute values was normalized to 1.0. A random delay with uniform distribution 30 time-unit points wide was given and centered at the time unit 20. A trace of 64 samples was used.

The techniques were investigated in the presence of noise. Two types of noise were generated: white and nonwhite (Fig.2). To accomplish this, zero-mean Gaussian random numbers with a standard deviation σ = 0.2 were generated and superimposed to the original signal sequence using different values of SNR's. Nonwhite noise was generated by filtering the white noise sequence using the following 11-point filter [17]:

$$y_n = \frac{1}{429}\sum_{i=-5}^{5} w_i q_{n+i} \quad (27)$$

where q_n is the nth random number, y_n is the nth output of the filter, $w_i = w_{-i}$, $w_0 = 89$, $w_1 = 84$, $w_2 = 69$, $w_3 = 44$, $w_4 = 9$, and $w_5 = -36$.

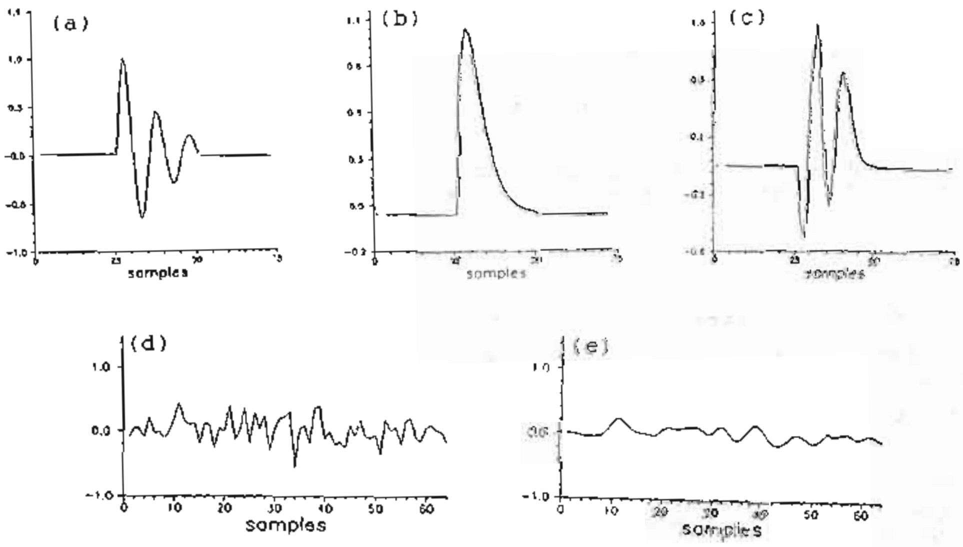

Fig.2 Original signals used in simulations of computer-generated data (a) Signal $s_1(t)$, (b) Signal $s_2(t)$, (c) Signal $s_3(t)$, (d) White noise, and (e) Coloured noise

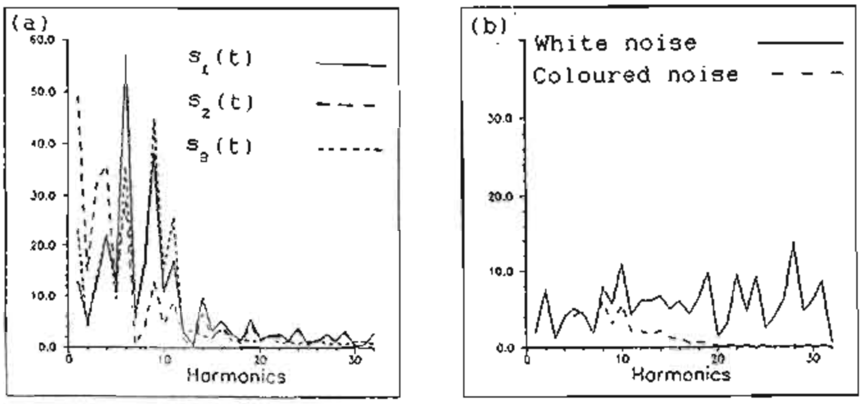

Fig.3 (a) The Fourier spectra of $s_1(t)$, $s_2(t)$, $s_3(t)$
(b) The Fourier spectra of white and nonwhite noise

Fig.3 shows the Fourier amplitudes of $s_1(t)$, $s_2(t)$, $s_3(t)$, the white noise sequence and the nonwhite noise sequence. The simulations were performed for three various signal-to-noise ratios (SNR_i's). For each SNR value, 50 ensembles (each consists of 50 noisy signals) were generated. The SNR_i here is defined as follows:

$$SNR_i = \frac{1}{M} \sum_{i=1}^{M} \left\{ \sqrt{\sum_{k=0}^{24} s^2(k) / \sum_{j=0}^{63} n^2(j)} \right\} \qquad (28)$$

where M is the number of noisy signals to be averaged. s(k) represents the sampled curve in Fig.2, and n(j) represents the noise.

IV. RESULTS

The seven waveform estimation techniques outlined in Sec. II have been applied to the simulated ensembles of the three signals generated in the way described in the previous section. Figures 4-6 give examples of the estimated signals for a typical SNR value. In the figures, the position of the estimated waveforms are abitrary shifted. From the figures it is clear that technique EA fails completely to retrieve the three waveforms $s_1(t)$, $s_2(t)$ and $s_3(t)$, while technique AICF recovers successfully the three waveforms. Table I shows some numerical results for $s_1(t)$, $s_2(t)$, and $s_3(t)$ corrupted with white and nonwhite noise. The results consist of the mean values of the 50 individual normalized root-mean-squared errors (RMSE). It is calculated from

$$RMSE = \frac{\sqrt{\sum_{k=0}^{18} \bar{y}^2(k-19) + \sum_{k=19}^{43} [s(k-19) - \hat{y}(k-19)]^2 + \sum_{k=44}^{63} \bar{y}^2(k-19)}}{\sqrt{\sum_{k=0}^{24} s^2(k)}} \qquad (29)$$

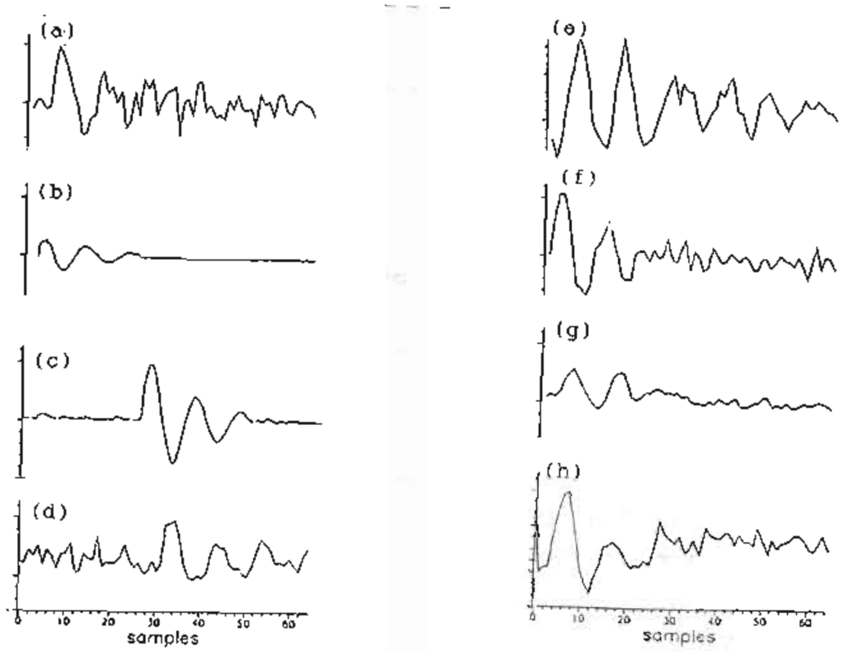

Fig.4 Noisy signal $s_1(t)$ and the recovered waveforms. (a) The signal with additive white noise (SNR = 0 dB), (b) recovered signal using ensemble averaging, (c) using coherent averaging, (d) using Woody's technique, (e) using weighted averaging, (f) using adaptive impulse correlated filter, (g) using spectral averaging, (h) using bispectral averaging.

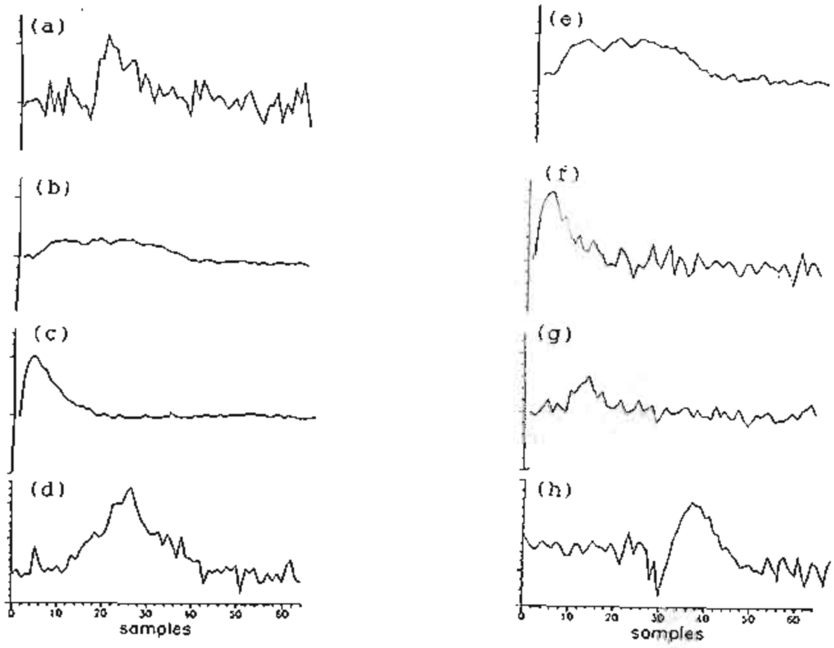

Fig.5 Noisy signal $s_2(t)$ and the recovered waveforms. The same captions as in Fig.4.

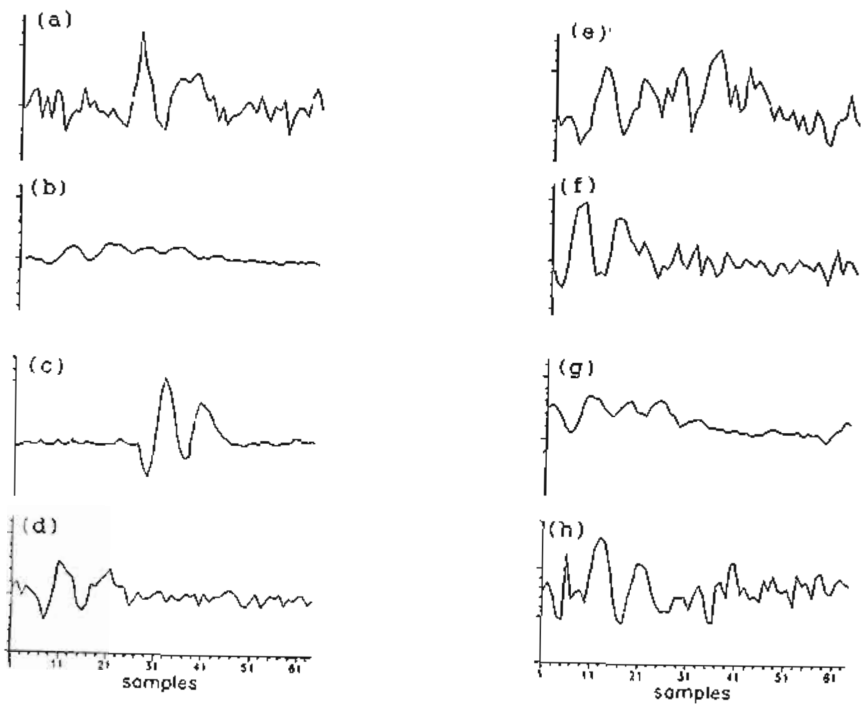

Fig.6 Noisy signal $s_3(t)$ and the recovered waveforms. The same captions as in Fig.4.

where $\hat{y}(k)$ denotes the recovered signal and is arbitrary shifted in time so that a cross correlation function has a maximum value. The cross correlation function here is in the form of

$$\Phi(\tau) = \sum_{k=0}^{24} s(k)\, \hat{y}(k+\tau), \qquad \tau = 0, 1, 2, \ldots, 127 \tag{30}$$

In order to quantify the performance of one technique relative to another, two well known SNR estimators were used. These estimators compute an estimate of the SNR of a pair of noise corrupted signals x_1 and x_2, that have been sampled n times [18,19]. The first is the so-called maximum likelihood estimator, given by

$$SNR_M = \frac{2\sum_{i=1}^{N} x_{1i}\, x_{2i}}{\sum_{i=1}^{N} (x_{1i} - x_{2i})^2} \tag{31}$$

where x_{1i} and x_{2i} are the ith samples of x_1 and x_2, respectively. This estimator is asymptotically Gaussian for finite N [18]. The second estimator that was used is based on the sample correlation coefficient between x_1 and x_2:

Table I Results of the mean values of RMSE obtained from 50 sets of noisy curves $s_1(t)$, $s_2(t)$ and $s_3(t)$ for various values of SNR's (white and coloured noise)

Noise	Tech-nique Type	$SNR_i = 2$ (3 dB)			$SNR_i = 1$ (0 dB)			$SNR_i = 0.5$ (-3 dB)		
		$s_1(t)$	$s_2(t)$	$s_3(t)$	$s_1(t)$	$s_2(t)$	$s_3(t)$	$s_1(t)$	$s_2(t)$	$s_3(t)$
WHITE	EA	0.46	-	-	-	-	-	0.49	-	-
	CA	0.04	0.08	0.05	0.04	0.07	0.05	0.15	0.29	0.17
	WT	0.22	0.37	0.15	0.27	0.45	0.23	-	-	0.45
	WA	0.33	0.39	0.26	0.33	-	0.29	0.45	-	0.50
	AICF	0.15	0.28	0.19	0.14	0.24	0.18	0.39	-	0.48
	SA	0.25	0.43	0.37	0.35	-	0.45	-	-	0.34
	BA	0.19	0.35	0.28	0.32	0.38	0.38	-	-	0.40
NONWHITE	EA	0.37	-	0.42	-	-	-	0.46	-	-
	CA	0.03	0.05	0.02	0.04	0.07	0.05	0.09	0.19	0.12
	WT	0.22	0.41	0.26	0.24	0.49	0.23	-	-	-
	WA	0.39	-	0.30	0.32	-	0.39	0.39	-	0.37
	AICF	0.09	0.18	0.12	0.14	0.14	0.18	0.19	0.20	0.17
	SA	0.35	-	0.43	0.35	-	0.45	0.37	-	-
	BA	0.23	0.21	0.41	0.23	0.30	0.38	0.43	-	-

- indicates failure in the signal recovery processes

$$r = \frac{\sum_{i=1}^{N} x_{1i}\, x_{2i}}{\sqrt{\sum_{i=1}^{N} x_{1i}^2 \sum_{j=1}^{N} x_{2j}^2}} \qquad (32)$$

The SNR estimator is then given by

$$SNR_r = A\,\frac{r}{1-r} + B \qquad (33)$$

where the constants

$$A = \exp\left(\frac{-2}{N-3}\right) \quad \text{and} \quad B = -\frac{1}{2}\left[1 - \exp\left(\frac{-2}{N-3}\right)\right] \qquad (34)$$

The constants A and B make SNR_r unbiased for finite. through large values of N (19). In each case. the SNR estimates were computed for N = 64. These SNR estimates were computed from all waveform estimates and subsequently averaged to form a more stable measure. The results are tabulated in Tables II and III.

In the signal estimation processes for the 50 ensembles of data, if the RMSE value became more than 0.5 and the value of the estimated SNR was less than that of the original noisy signal, then it was considered that the signal recovery failed. SNR's values greater than 10 were considered as successful recovery and are denoted by (*).

The results in tables I-III reveal the following:

1) Regarding the signals contaminated with white noise, the CA approach demonstrates the best performance in recovering the true signals $s_1(t)$, $s_2(t)$ and $s_3(t)$. The AICF technique shows better performance than other

Table II Results of the mean values of the SNR_M estimates obtained from 50 sets of noisy curves $s_1(t)$, $s_2(t)$ and $s_3(t)$ for various values of SNR's (white and coloured noise)

Noise	Technique Type	SNR_i = 2 (3 dB)			SNR_i = 1 (0 dB)			SNR_i = 0.5 (-3 dB)		
		$s_1(t)$	$s_2(t)$	$s_3(t)$	$s_1(t)$	$s_2(t)$	$s_3(t)$	$s_1(t)$	$s_2(t)$	$s_3(t)$
WHITE	EA	2.82	-	-	-	-	-	2.11	0.99	1.64
	CA	•	•	•	•	•	•	•	•	•
	WT	3.66	3.37	2.42	3.83	1.45	3.28	4.13	1.77	3.03
	WA	4.64	-	3.39	7.98	1.33	6.16	3.01	1.13	7.82
	AICF	•	•	•	•	•	•	•	5.38	•
	SA	2.83	-	2.96	2.97	-	3.05	2.94	0.99	4.29
	BA	5.13	1.73	2.51	•	2.28	2.36	2.87	2.33	5.06
NONWHITE	EA	2.71	-	2.09	1.58	-	-	2.99	0.91	2.35
	CA	•	•	•	•	•	•	•	•	•
	WT	3.65	2.92	2.82	4.98	3.55	3.28	3.10	1.97	3.23
	WA	3.37	-	3.62	3.85	1.33	3.55	2.47	0.97	2.54
	AICF	•	•	•	•	•	•	•	7.46	•
	SA	2.77	-	2.91	5.54	-	3.14	2.94	0.89	3.69
	BA	5.62	2.27	2.13	5.81	2.28	2.36	2.78	1.56	2.74

• indicates SNR's values greater than 10.
– indicates SNR's values less than that of the original noisy signals.

techniques. The RMSE values show failures in signal recovery process of $s_2(t)$ for all techniques except the CA (for SNR = -3dB). Furthermore, techniques WT, SA and BA fail in the recovery of the three signals when SNR = - 3 dB. As for the SNR_M and SNR_r values, techniques WT, WA and BA offered higher values than those obtained by the SA approach. Technique EA possesses the worst performance.

2) Comparing the RMSE values obtained for the case of coloured noise, the CA technique shows also the best performance. Here, again the AICF demontrates better results than other techniques. Failures in the signal recovery of the signal $s_2(t)$ happen in both WA and SA approaches even for SNR = 3 dB. Techniques WT, SA and WA show better values of the SNR_M and SNR_r estimates than SA. Again, EA shows the worst performance.

From Tables I-III and Figures 4-6, it can be seen that the AICF approach can recover a signal waveform with recognizable features for the cases that the jitter was severe enough to obscure the signal and the signal-to-noise ratio is relatively low (white or coloured noises).

From the point of view of programming the techniques, the AICF is simpler than WT, WA, SA and BA. Also, computation time of AICF is faster.

V. DISCUSSION

The performance of seven techniques of improving the signal-to noise ratio of bioelectric signals with variable latency has been numerically investigated using computer-generated signals and noises. The numerical values have shown that the technique based on adaptive

Table III Results of the mean values of the SNR_r estimates obtained from 50 sets of noisy curves $s_1(t)$, $s_2(t)$ and $s_3(t)$ for various values of SNR's (white and coloured noise)

Noise	Technique Type	SNR_i = 2 (3 dB)			SNR_i = 1 (0 dB)			SNR_i = 0.5 (-3 dB)		
		$s_1(t)$	$s_2(t)$	$s_3(t)$	$s_1(t)$	$s_2(t)$	$s_3(t)$	$s_1(t)$	$s_2(t)$	$s_3(t)$
WHITE	EA	2.21	-	-	-	-	-	2.60	0.91	2.69
	CA	*	*	*	*	*	*	*	*	*
	WT	3.94	2.04	2.63	4.10	1.12	2.97	4.45	1.75	2.51
	WA	2.53	-	2.63	7.76	1.22	6.43	2.46	1.02	4.05
	AICF	*	*	*	*	*	*	*	7.22	9.28
	SA	2.29	-	2.61	2.49	-	2.79	2.11	0.94	2.86
	BA	5.52	1.63	2.33	*	2.19	2.36	2.23	1.64	4.72
NONWHITE	EA	2.26	-	2.46	-	-	-	2.32	0.83	2.64
	CA	*	*	*	*	*	*	*	*	*
	WT	3.96	2.29	2.62	4.90	3.28	2.97	2.73	1.74	3.67
	WA	2.48	-	3.39	2.86	1.22	3.80	3.37	1.51	3.62
	AICF	*	*	*	*	*	*	*	5.10	*
	SA	2.24	-	2.55	4.35	-	2.79	2.15	0.83	2.83
	BA	5.05	2.15	2.18	6.15	2.19	2.03	2.84	1.64	2.02

* indicates SNR's values greater than 10.
\- indicates SNR's values less than that of the original noisy signals.

impulse correlated filter (AICF) was the most efficient technique for recovering a signal with recognizable features even if the SNR is relatively low and the signal is embedded in white or nonwhite noise.

Although coherent averaging technique gives smaller RMSE values and higher SNR's values, synchronization of the different waveforms to a stable time reference is difficult for biological signals for two reasons. First, an invariant fiducial point cannot be found on any waveform because they continually vary in time. Second, the time between fiducial points is variable; even if the fiducial points were invariant and could be found with perfect accuracy each cycle, there is no assurance that the signals will be stable with that reference. Moreover, coherent averaging needs a large number of records to obtain a good estimation of the signal, and cannot show eventual dynamic variations of the signal shape.

The three techniques of Woody, weighted averaging and bispectral averaging present a better performance than technique of spectral averaging. The improvement resulting from the techniques of spectral and bispectral averaging is hardly justified by the increase in computational complexity associated with their implementation.

These results will be very useful for recovering low-amplitude event-related vioelectric signals embedded in white or nowhite noise.

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